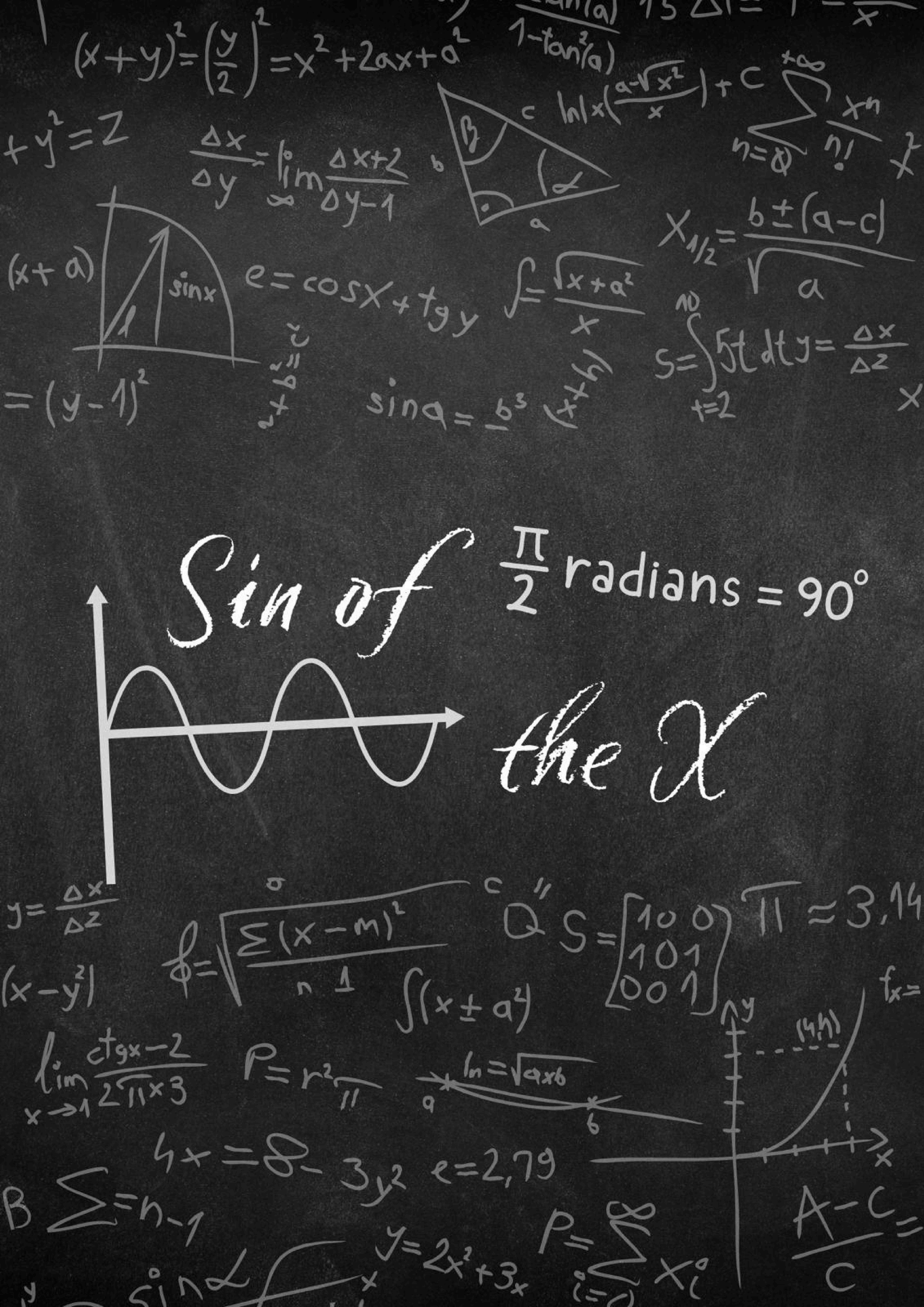
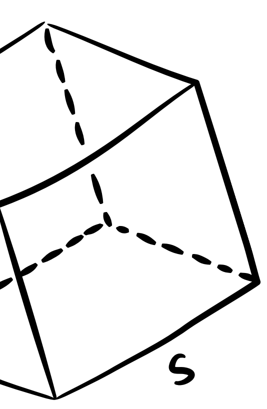
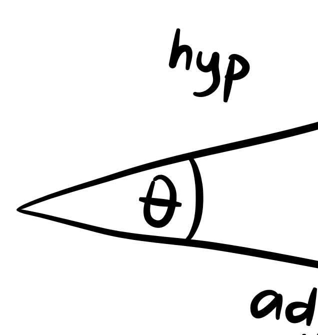

Sin of $\frac{\pi}{2}$ radians = 90°
the X

CONTENTS

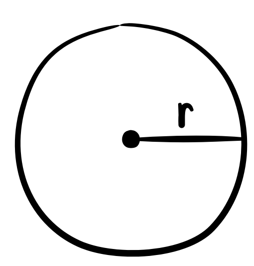




$\cos(\theta) =$

$V = s^3$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$



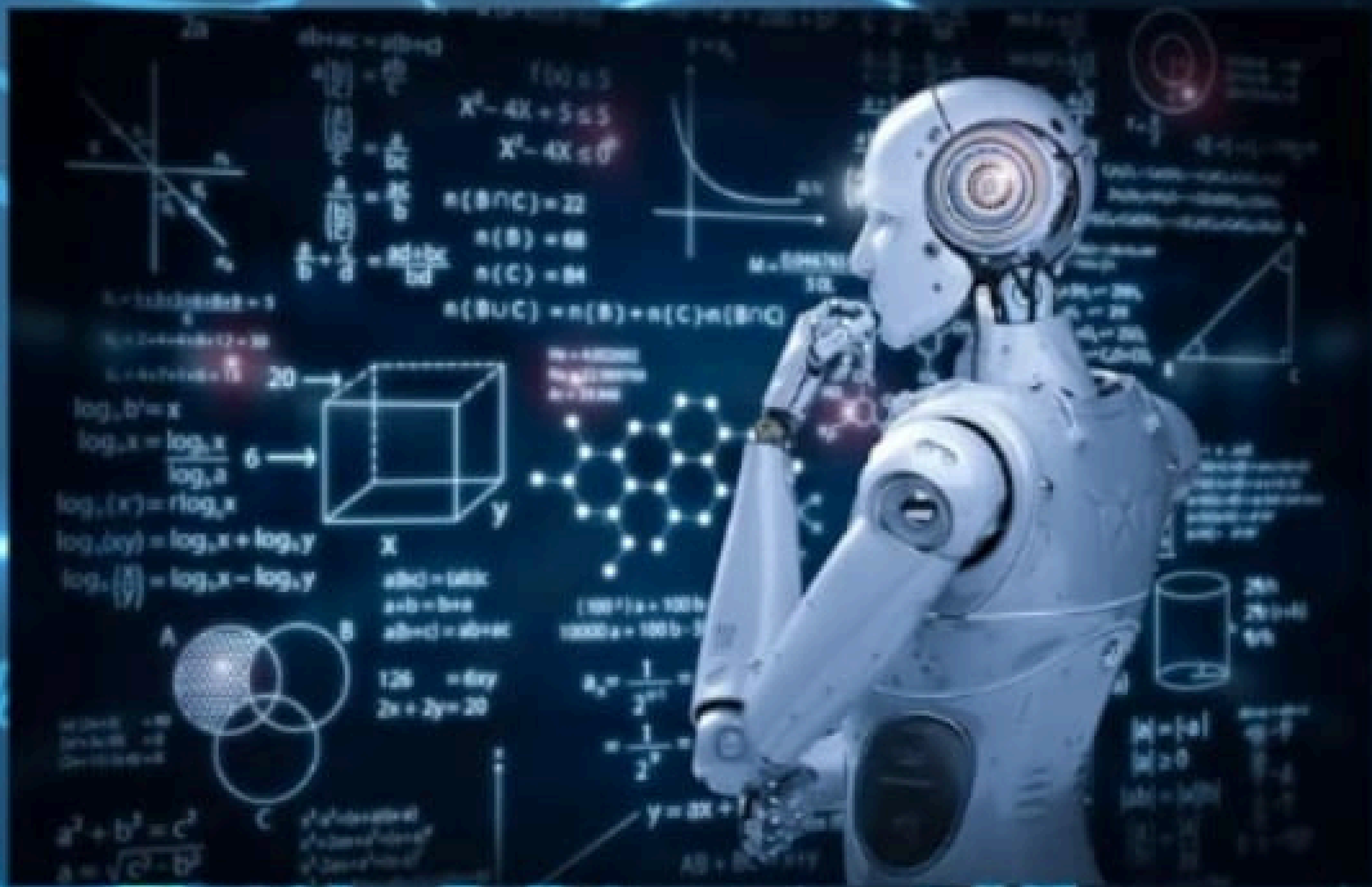
$A = \pi r^2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

MACHINE LEARNING: THE MATHS!

ARYA P

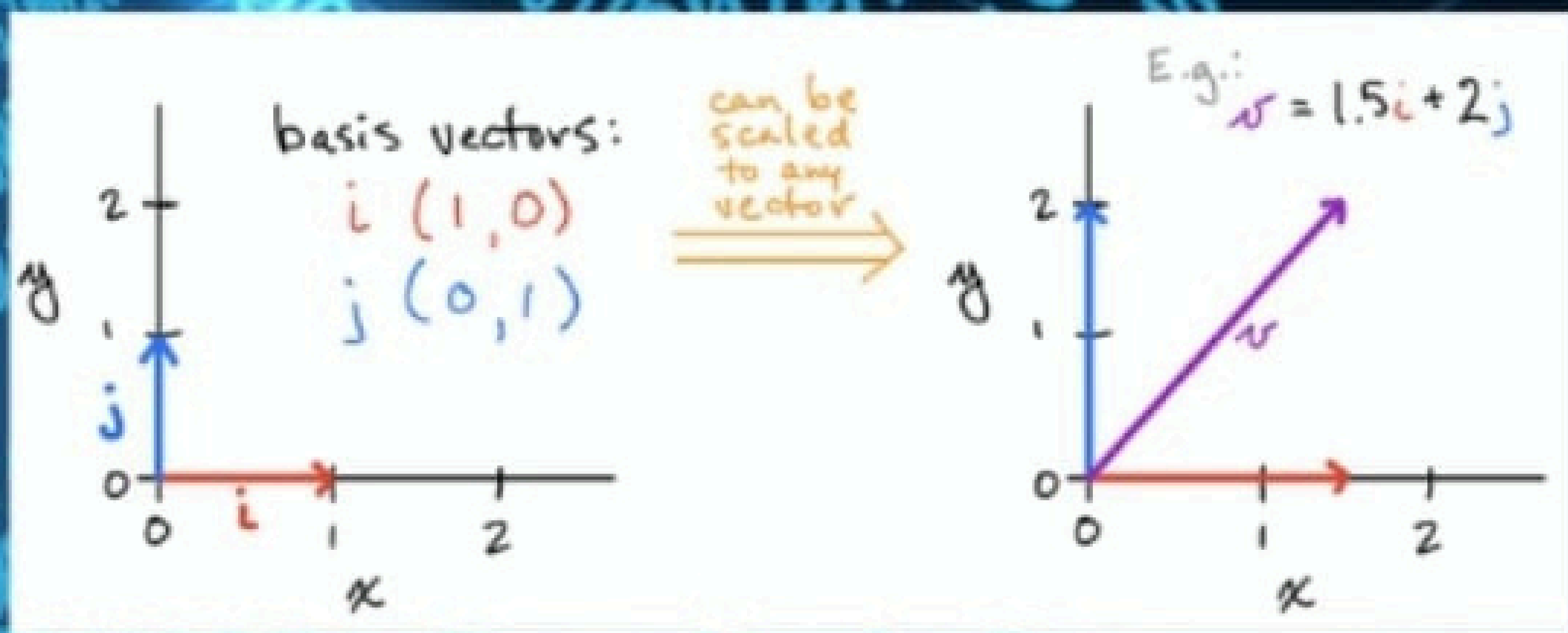
WHAT IS MACHINE LEARNING?

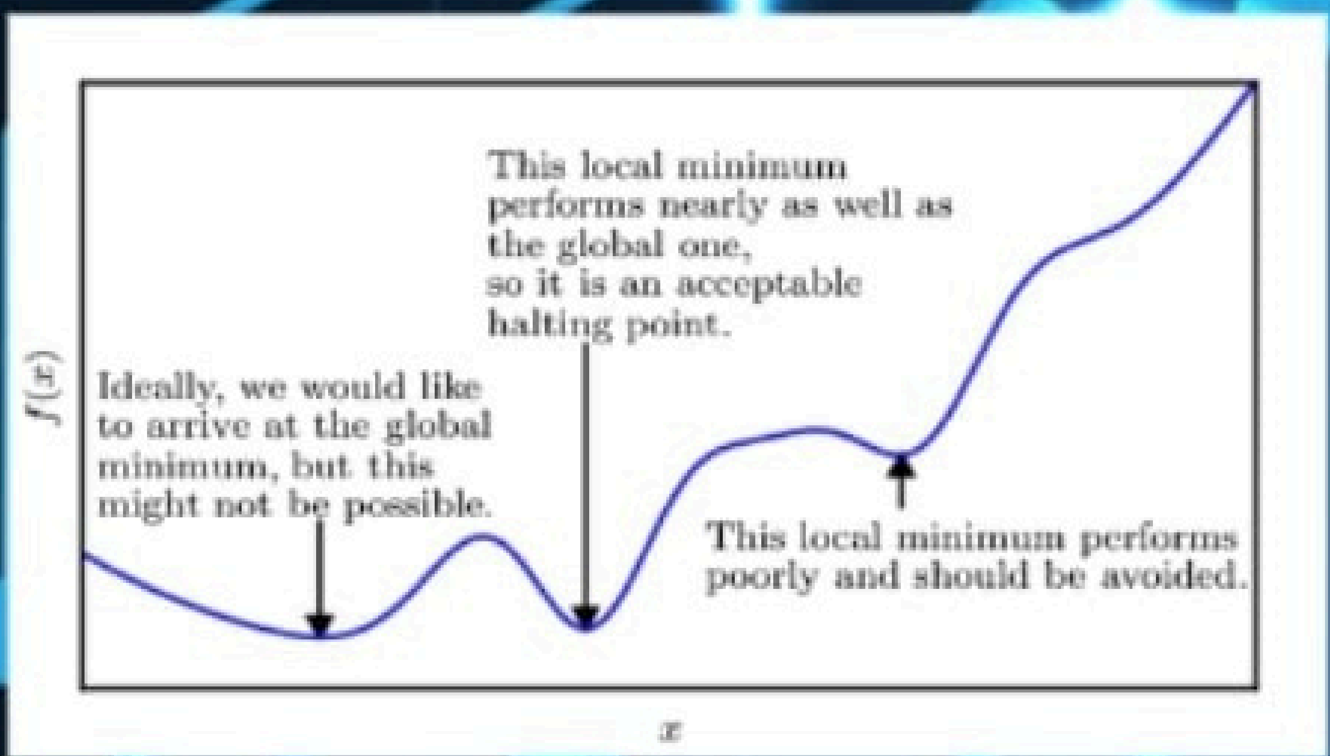
Machine learning is a branch of artificial intelligence that empowers computers to learn from and make decisions based on DATA. At its core, machine learning uses various mathematical concepts, such as linear algebra, calculus, and probability AND MORE - all of which you can read about below...



HOW LINEAR ALGEBRA IS USED

Linear algebra is essential for handling and manipulating data represented as vectors and matrices, which form the basis of many machine learning algorithms. On the side, you can see an example of linear algebra, which is utilised by machine learning!



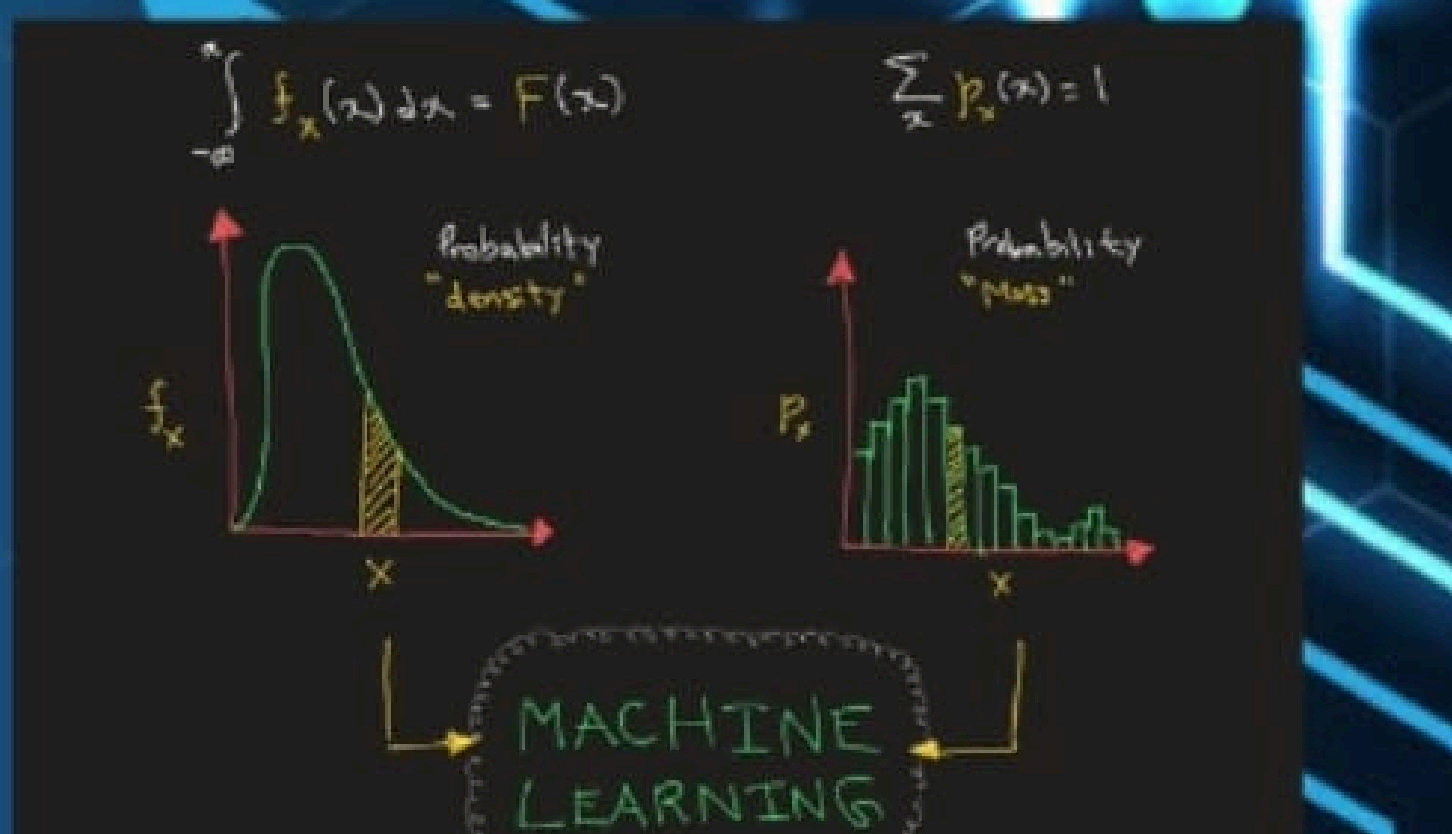


HOW CALCULUS IS USED

Calculus is used to optimize these algorithms, particularly in training neural networks where gradient descent methods minimise error functions. The graph on the left showcases how machine learning minimises error functions from the local minimum.

PROBABILITY AND STATISTICS

Now, probability and statistics are fundamental for making inferences from data, estimating distributions and handling uncertainty. Using probability density (as shown to the right), AI can learn and predict future patterns that would potentially assist us on decisions we make - all just through probability!



CRYPTOGRAPHY

AND ITS MATHEMATICAL FOUNDATIONS

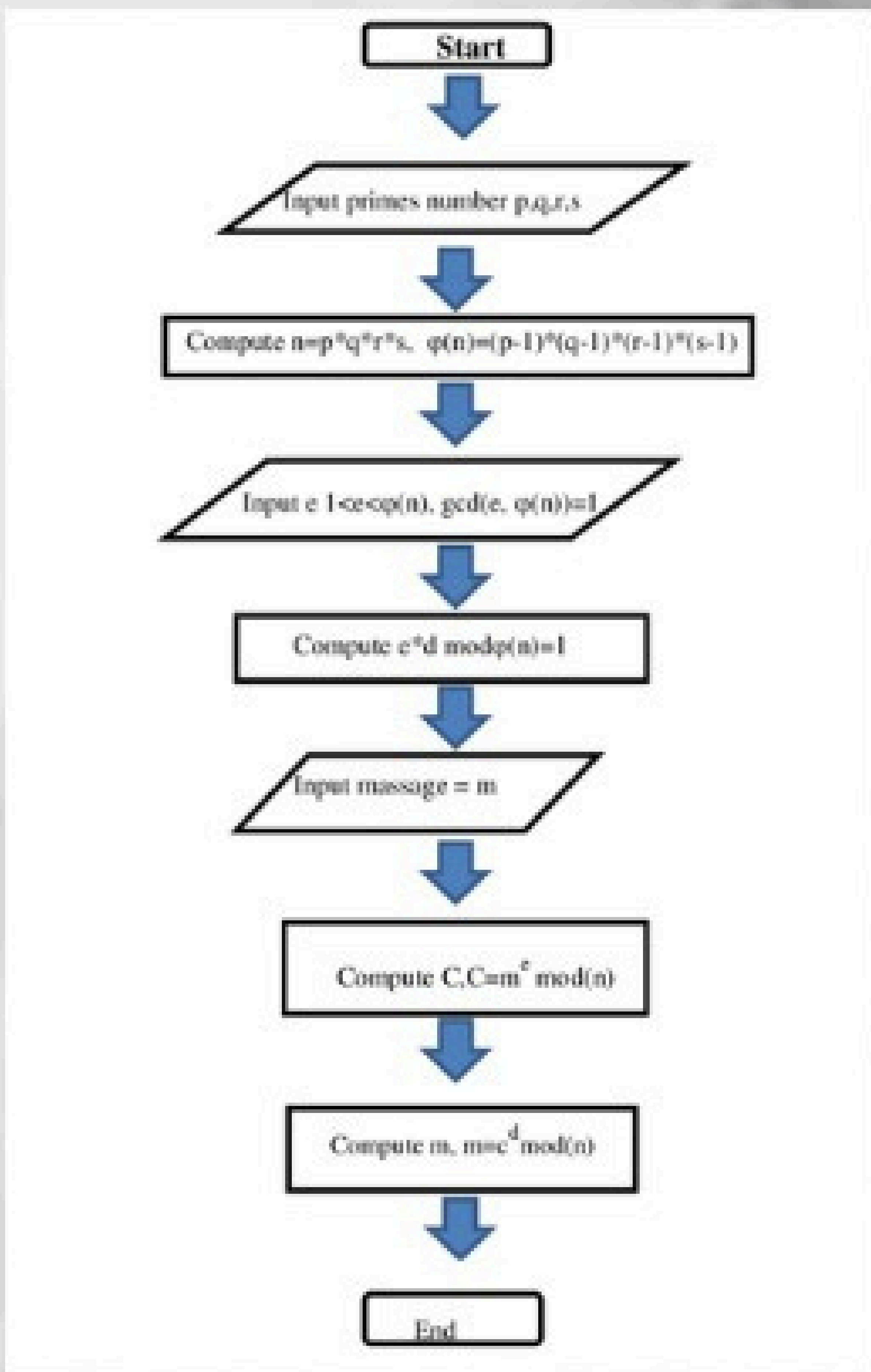
ARYA P

WHAT IS CRYPTOGRAPHY?

Cryptography is the practice and study of techniques for securing communication and data from adversaries. It involves various mathematical concepts and is fundamental to modern computer society.

NUMBER THEORY, AND THE RSA ALGORITHM

Number theory is a branch of pure mathematics devoted to the study of integers and integer-valued functions. It encompasses various properties and relationships of numbers. In particular, I'm going to focus on the RSA algorithm! The "RSA algorithm" stands for the Rivest-Shamir-Adleman algorithm, which uses a pair of public and private keys for encryption and decryption. As you can see in the diagram to the right, prime factorisation is used as large prime numbers are selected and multiplied to contain what we'll call "n". The "totient" function ϕ(n) is calculated, after which an encryption (e) and decryption key (d) are computed that have to meet requirements. The public key is (e,n) whilst the private key is (d,n). After that, a message which we can call "m", is encrypted as C=M^e (so mod(n)) and decrypted as C= M^d.



THE RIEMANN HYPOTHESIS

The Riemann hypothesis is an incredibly famous problem in mathematics, proposed by Bernhard Riemann, a German mathematician in 1859. It said that all non-trivial zeros of a function called the "Riemann zeta function" have their real parts equal to 1/2. The confusing part? While a lot of numerical evidence supports it, an actual formal proof has yet to be solved, and has remained a mystery.

Trevor L

Proving 1=2 (one equals two) by Trevor Lee (12W)

Today on The Sin of x, I am going to show all of you that 1=2. Without further ado, let's start!

Proof I : Numeric Proof

$1=2$ } ×2 _ (1)

$2=4$ } −3 _ (2)

$-1=1$ } square _ (3)

$1=1$ _ (4)

∵ (4) is true

∴ (1) must also be true

['∵' means 'because'
'∴' means 'therefore']

Easy-peasy, isn't it? Okay, I see that you are still unconvinced, so allow me to show you another proof –

Proof II : Algebraic Proof

Let two variables x and y such that $x=y$. _ (1)

∴ $x=y$ } ×x _ (2)

$x^2=xy$ } $-y^2$ _ (3)

$x^2-y^2=xy-y^2$ _ (4)

$(x+y)(x-y)=y(x-y)$ _ (5)

$x+y=y$ } sub $y=x$ _ (6)

$2x=x$ } ÷x _ (7)

$2=1$ ['sub' stands for 'substitute'] _ (8)

I am sure you are convinced now. What? You are still not? Don't worry, I have six more proofs to show you!

Proof III : Derivative Proof

$x^2 = x \times x$ _ (1)

$x^2 = \overbrace{x+x+x+\ldots+x}^{x \text{ times}}$ } differentiate _ (2)

power rule { $\frac{d}{dx}x^2 = \frac{d}{dx}\overbrace{(x+x+x+\ldots+x)}^{x \text{ times}}$ } sum rule _ (3)

$2x = \overbrace{1+1+1+\ldots+1}^{x \text{ times}}$ _ (4)

$2x = 1 \times x$ } ÷x _ (5)

$2=1$ _ (6)

['power rule' : $\frac{d}{dx}x^n = nx^{n-1}$
'sum rule' : $\frac{d}{dx}(f(x)+g(x)) = f'(x)+g'(x)$]

Proof IV : Integration Proof

$\int \frac{1}{x \ln x}dx = \int \frac{1}{\ln x} \times \frac{1}{x}dx$ _ (1)

Let $u=\frac{1}{\ln x}$, $dv=\frac{1}{x}dx$ ['du' can be _ (2)

$\Rightarrow du = -\frac{1}{x\ln^2 x}$, $v=\ln x$ obtained by chain rule] _ (3)

∵ $\int u\,dv = uv - \int v\,du$ [integration by part] _ (4)

∴ $\int \frac{1}{x\ln x}dx = \frac{1}{\ln x} \times \ln x - \int \ln x \times \left(-\frac{1}{x\ln^2 x}\right)dx$ _ (5)

$= 1 - \int -\frac{1}{x \ln x}dx$ _ (6)

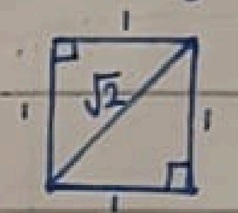
$\Rightarrow \int \frac{1}{x\ln x}dx = 1 + \int \frac{1}{x \ln x}dx$ } $-\int \frac{1}{x\ln x}dx$ _ (7)

$0=1$ } +1 _ (8)

$1=2$ ['⇒' means 'therefore'] _ (9)

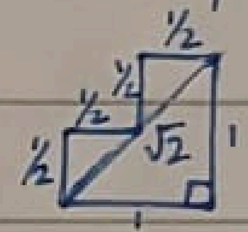
Proof V : Limit Proof

Get a square with side length of 1



_ (1)

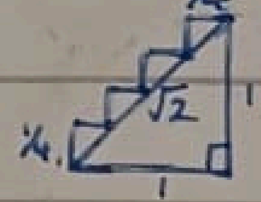
Cut the top left corner as shown



_ (2)

As shown in diagram, cutting the corner does not change the length of the 'stairs' – it remains as 2 _ (3)

Repeat (2) as shown:



_ (4)

Repeat ad infinitum, at the limit, the 'stairs' will be identical to the diagonal line _ (5)

['ad infinitum' is fancy Latin for 'for ever, without ending']

∴ At the limit, length of 'stairs' = length of the diagonal _ (6)

∴ $2=\sqrt{2}$ } square _ (7)

$4=2$ } ÷2 _ (8)

$2=1$ _ (9)

Continue at the next page

Proof VI : Logarithmic Proof

$1^1 = 1^2$ } log base 1 — (1)

$\log_1(1^1) = \log_1(1^2)$ } power rule [$y=\log_a(b) \Longleftrightarrow a^y=b$] — (2)

$1\log_1(1) = 2\log_1(1)$ ['$\Longleftrightarrow$' means 'equivalent'] — (3)

$1 \times 1 = 2 \times 1$ — (4)

$1 = 2$ ['power rule': $\log_a(b^x) = x\log_a(b)$] — (5)

Proof VII : Rational Number Proof

Define a new operation '$\Diamond$' over the rational number such that $\frac{a}{b} \Diamond \frac{c}{d} = \frac{ab+cd}{bd}$; $a,b,c,d \in \mathbb{Z}$; $b,d \neq 0$ — (1)

['$x,y \in \mathbb{Z}$' means 'x and y are both integers'
'$\in$' means 'part of'; '$\mathbb{Z}$' means 'the set of integers']

$\frac{1}{2} \Diamond \frac{1}{2} = \frac{1\times2+1\times2}{2\times2}$ — (2)

$= \frac{2+2}{4}$ — (3)

$= \frac{4}{4}$ — (4)

$= 1$ — (5)

$\frac{1}{2} \Diamond \frac{2}{4} = \frac{1\times2+2\times4}{2\times4}$ — (6)

$= \frac{2+8}{8}$ — (7)

$= \frac{10}{8}$ — (8)

$= \frac{5}{4}$ — (9)

$\because \frac{1}{2} = \frac{2}{4}$ — (10)

$\therefore \frac{1}{2} \Diamond \frac{1}{2} = \frac{1}{2} \Diamond \frac{2}{4}$ — (11)

$\therefore 1 = \frac{5}{4}$ } $\times 4$ — (12)

$4 = 5$ } -3 — (13)

$1 = 2$ — (14)

Proof VIII : Complex Number Proof

$\frac{-1}{1} = \frac{1}{-1}$ — (1)

$\sqrt{\frac{-1}{1}} = \sqrt{\frac{1}{-1}}$ — (2)

$\frac{\sqrt{-1}}{\sqrt{1}} = \frac{\sqrt{1}}{\sqrt{-1}}$ — (3)

$\frac{i}{1} = \frac{1}{i}$ } $\div 2$ ['i' stands for $\sqrt{-1}$] — (4)

$\frac{i}{2} = \frac{1}{2i}$ } $+\frac{3}{2i}$ — (5)

$\frac{i}{2} + \frac{3}{2i} = \frac{1}{2i} + \frac{3}{2i}$ } $\times i$ — (6)

$i\left(\frac{i}{2} + \frac{3}{2i}\right) = i\left(\frac{1}{2i} + \frac{3}{2i}\right)$ — (7)

$\frac{i^2}{2} + \frac{3i}{2i} = \frac{i}{2i} + \frac{3i}{2i}$ — (8)

$\frac{-1}{2} + \frac{3}{2} = \frac{1}{2} + \frac{3}{2}$ — (9)

$1 = 2$ // Q.E.D. — (10)

['Q.E.D.' stands for 'quod erat demonstradum'. (again)
fancy Latin for 'which was to be shown']

So these are all eight proofs of 1=2. (I will admit that most are from the Internet) Any how, you should now all believe that 1=2. Thank you for reading!

~ The End ~

Outro: Obviously, all of these proofs have flaws in them. (Sadly, Mathematics is still intact) If you are wondering why you may want to understand these wrong proofs, it is because spotting where a proof went wrong can help you spot mistakes easier. After all, not all mistakes lead to obvious contradictions like 1=2.

For more information about the proofs, feel free to ask your Maths teacher. I am sure they will be happy to answer (hopefully).

Understanding how $e^{i\theta} = \cos\theta + i\sin\theta$ (by Trevor Lee 12W)

Prior knowledge required: imaginary and complex numbers, limits of functions, differentiation (for real numbers), radians

Chapter I: Derivative of complex numbers

For $z, z_1 \in \mathbb{C}$, $f'(z_1) = \lim_{z \to z_1} \frac{f(z)-f(z_1)}{z-z_1}$. By defining $\Delta z = z - z_1$, the above can be written as

$$f'(z_1) = \lim_{\Delta z \to 0} \frac{f(z_1+\Delta z)-f(z_1)}{\Delta z}$$

Chapter II: The Taylor Series and the Maclaurin Series

Assume a function can be written as a polynomial, such that

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \dots \quad -(1)$$

Let $a=x$, then $f(a) = c_0$ [Note that $(x-a)$ becomes zero]

Differentiating (1) gives:

$$f'(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + 4c_4(x-a)^3 + \dots \quad -(2)$$

Let $a=x$, $f'(a) = c_1$

Differentiating (2) gives:

$$f''(x) = 2c_2 + 6c_3(x-a) + 12c_4(x-a)^2 + 20c_5(x-a)^3 + \dots \quad -(3)$$

Let $a=x$ (again), $f''(a) = 2c_2 \rightarrow \frac{f''(a)}{2} = c_2$

Differentiating (3) gives:

$$f'''(x) = 6c_3 + 24c_4(x-a) + 60c_5(x-a)^2 + 120(x-a)^4 + \dots$$

Let $a=x$, $f'''(a) = 6c_3 \rightarrow \frac{f'''(a)}{6} = c_3 \rightarrow \frac{f'''(a)}{3!}\ c_3$

Repeat ad infinitum ['Ad infinitum' is latin for 'continue forever'], we will get the Taylor series:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

or $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$, where $f^{(n)}(x)$ stands for the n^{th} derivative of $f(x)$.

The Maclaurin Series is a special case of the Taylor Series where $a=0$, which is as follows:

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3!}x^3 + \dots = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$$

Chapter III: The general binomial expansion

Consider the function $(1+x)^a$, its Maclaurin expansion would be: $(1+x)^a = 1 + ax + \frac{a(a-1)}{2}x^2 + \frac{a(a-1)(a-2)}{3!}x^3 + \dots$

or $(1+x)^a = \sum_{k=0}^{\infty} \binom{a}{k} x^k$, where $\binom{a}{k} = \frac{a(a-1)(a-2)\dots(a-k+1)}{k!}$

Let $L = \lim_{n\to\infty} \left|\frac{\mu_{n+1}}{\mu_n}\right|$, where μ_n is the n^{th} term of the series.

$$\therefore L = \lim_{n\to\infty} \left|\frac{\binom{a}{n+1}x^{n+1}}{\binom{a}{n}x^n}\right| \gg \left[\lim_{n\to\infty} \left|\frac{a(a-1)\dots(a-n+1)(a-n)x^n x\, n!}{a(a-1)\dots(a-n)x^n(n+1)n!}\right|\right]$$

$$= \lim_{n\to\infty} \left|\frac{(a-n)x}{n+1}\right| \ll$$

As $n \to \infty$, $(a-n) \to -n$ and $(n+1) \to n$

$$\therefore L = \lim_{n\to\infty} \left|\frac{-nx}{n}\right|$$

$$L = |-x|$$

By the ratio test, if $L<1$, then the series converges absolutely [meaning that the sum would be finite]. So if $|x|<1$, $\sum_{k=0}^{\infty}\binom{a}{k}x^k$ [the binomial expansion] converges absolutely. Note that this applies regardless of the value of a.

Special Chapter: Ratio test's validity

Consider a real number r such that $L<r<1$ $\left[L = \lim_{n\to\infty}\left|\frac{\mu_{n+1}}{\mu_n}\right|\right]$

$\therefore |\mu_{N+b}| = L^b|\mu_N| < r^b|\mu_N|$ for any $n>N$ and $b>0$.

$$\therefore \sum_{b=N+1}^{\infty} |\mu_b| = \sum_{b=1}^{\infty} |\mu_{N+b}| < \sum_{b=1}^{\infty} r^b|\mu_N| = |\mu_N|\sum_{b=1}^{\infty} r^b$$

From Geometric series, Let $S = r + r^2 + r^3 + \dots$

$$S - rS = r + r^2 + r^3 + r^4 + \dots - r^2 - r^3 - r^4 - \dots$$

$$(1-r)S = r$$

$$S = \frac{1}{1-r} \rightarrow \sum_{b=N+1}^{\infty}|\mu_b| < |\mu_N|\sum_{b=1}^{\infty} r^b = |\mu_N|\frac{1}{1-r},$$ that is: if $L<1$, the serie always converges absolutely.

Chapter IV: Sine and cosine [all angles in radians]

Given that $\frac{d}{d\theta}\sin\theta = \cos\theta$ and $\frac{d}{d\theta}\cos\theta = -\sin\theta$

The Maclaurin expansion of $\sin\theta$ and $\cos\theta$ are:

$$\sin\theta = \sin 0 + \cos 0 \times \theta - \frac{\sin 0}{2}\theta^2 - \frac{\cos 0}{3!}\theta^3 + \frac{\sin 0}{4!}\theta^4 + \dots$$

$$= \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

$$\cos\theta = \cos 0 - \sin 0 \times \theta - \frac{\cos 0}{2}\theta^2 + \frac{\sin 0}{3!}\theta^3 + \frac{\cos 0}{4!}\theta^4 - \dots$$

$$= 1 - \frac{\theta^2}{2} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots,$$ which will be important later.

Continue at the next page.

Chapter I: e and e^x

Consider a bank that gives you 100% interest rate a year. By the end of the year, you will have 2 times the money. Now consider that bank giving you 50% interest rate every 6 months. By the end of the year, you will have $1.5^2 = 2.25$ times the money. Now consider (yet again) the bank gives you $33\frac{1}{3}\%$ interest rate every 4 months, you will have $(1\frac{1}{3})^3 = 2.3\dot{7}\dot{0}$ times the money by the end of year. What is the limit to this type of interest rate? Well simple: $\lim_{n\to\infty}(1+\frac{1}{n})^n$, which is approximately 2.7182818..., or Mathematicians call it 'e'.

e^x is therefore $\lim_{n\to\infty}(1+\frac{1}{n})^{nx}$. Now, we can use the binomial expansion on the expression as $\lim_{n\to\infty}\frac{1}{n}<1$, so:

$$e^x = \lim_{n\to\infty}\left(1 + nx\frac{1}{n} + \frac{nx(nx-1)}{2}\left(\frac{1}{n}\right)^2 + \frac{nx(nx-1)(nx-2)}{3!}\left(\frac{1}{n}\right)^3 + \dots\right)$$

As $n\to\infty$, $(nx-a)\to nx$, when a is any finite number.

$$\therefore e^x = \lim_{n\to\infty}\left(1 + nx\frac{1}{n} + \frac{(nx)^2}{2}\left(\frac{1}{n}\right)^2 + \frac{(nx)^3}{3!}\left(\frac{1}{n}\right)^3 + \dots\right)$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

Chapter II: the original expression.

$$LHS = e^{i\theta}$$

$$= 1 + i\theta + \frac{(i\theta)^2}{2} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^5}{5!} + \frac{(i\theta)^6}{6!} + \frac{(i\theta)^7}{7!} + \dots$$

$$= 1 + i\theta - \frac{\theta^2}{2} - i\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + i\frac{\theta^5}{5!} - \frac{\theta^6}{6!} - i\frac{\theta^7}{7!} + \dots$$

$$= \left(1 - \frac{\theta^2}{2} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots\right) + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots\right)$$

$$= \cos\theta + i\sin\theta = RHS.$$

$$\therefore e^{i\theta} \equiv \cos\theta + i\sin\theta \text{ // Q.E.D.}$$

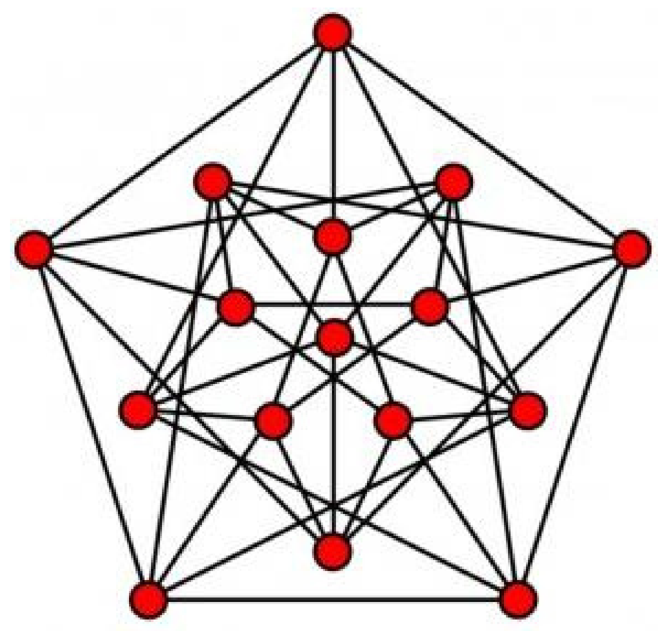
[Q.E.D. is short for 'Quod Erat Demonstradum', Latin for 'which was to be shown']

Benjamin E

Graph Theory

An Introduction to graph theory.

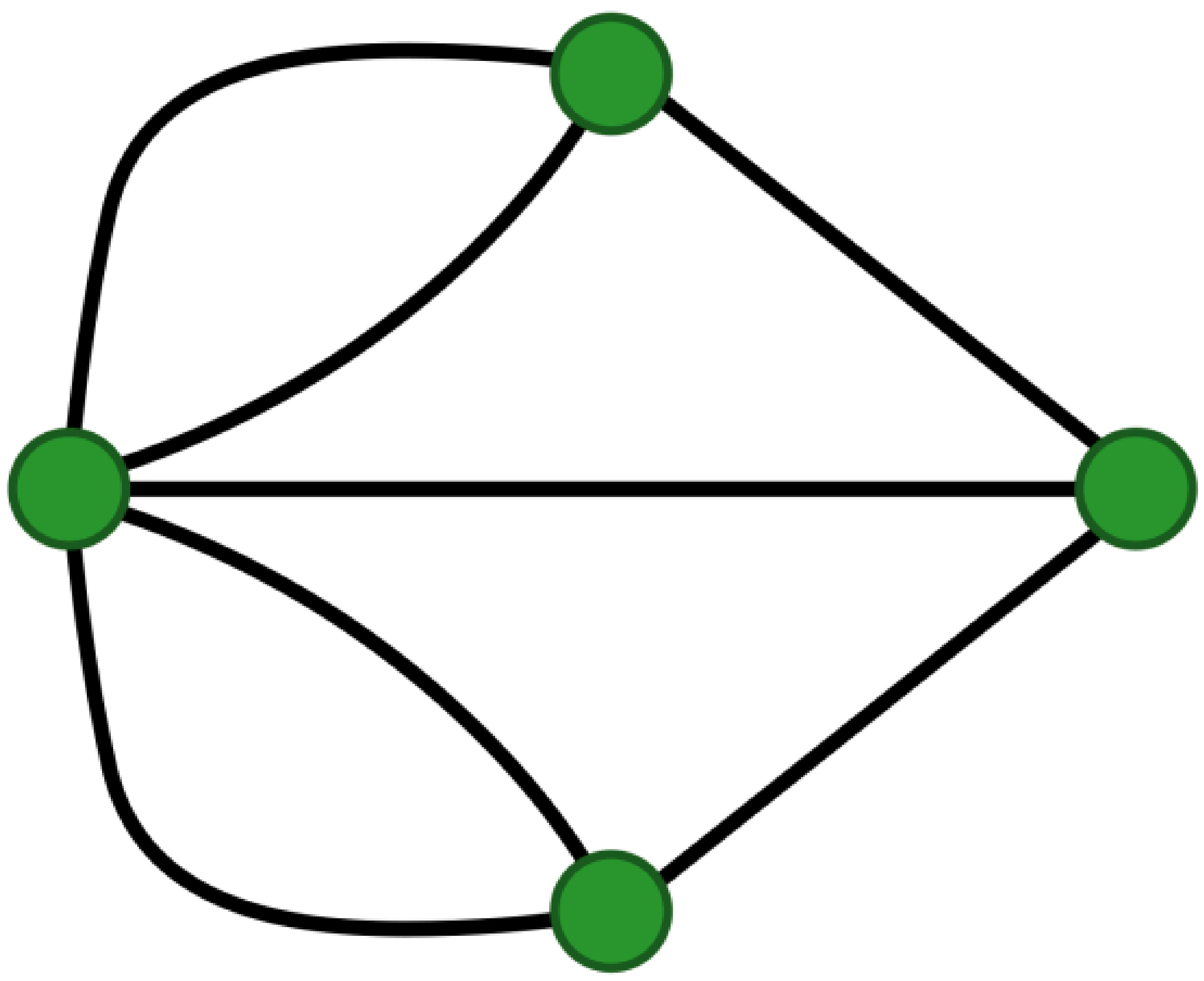
Graph theory in simple terms is the study of graphs which consist of points or vertices and their connections or edges. A path is defined as a series of vertices connected by edges.

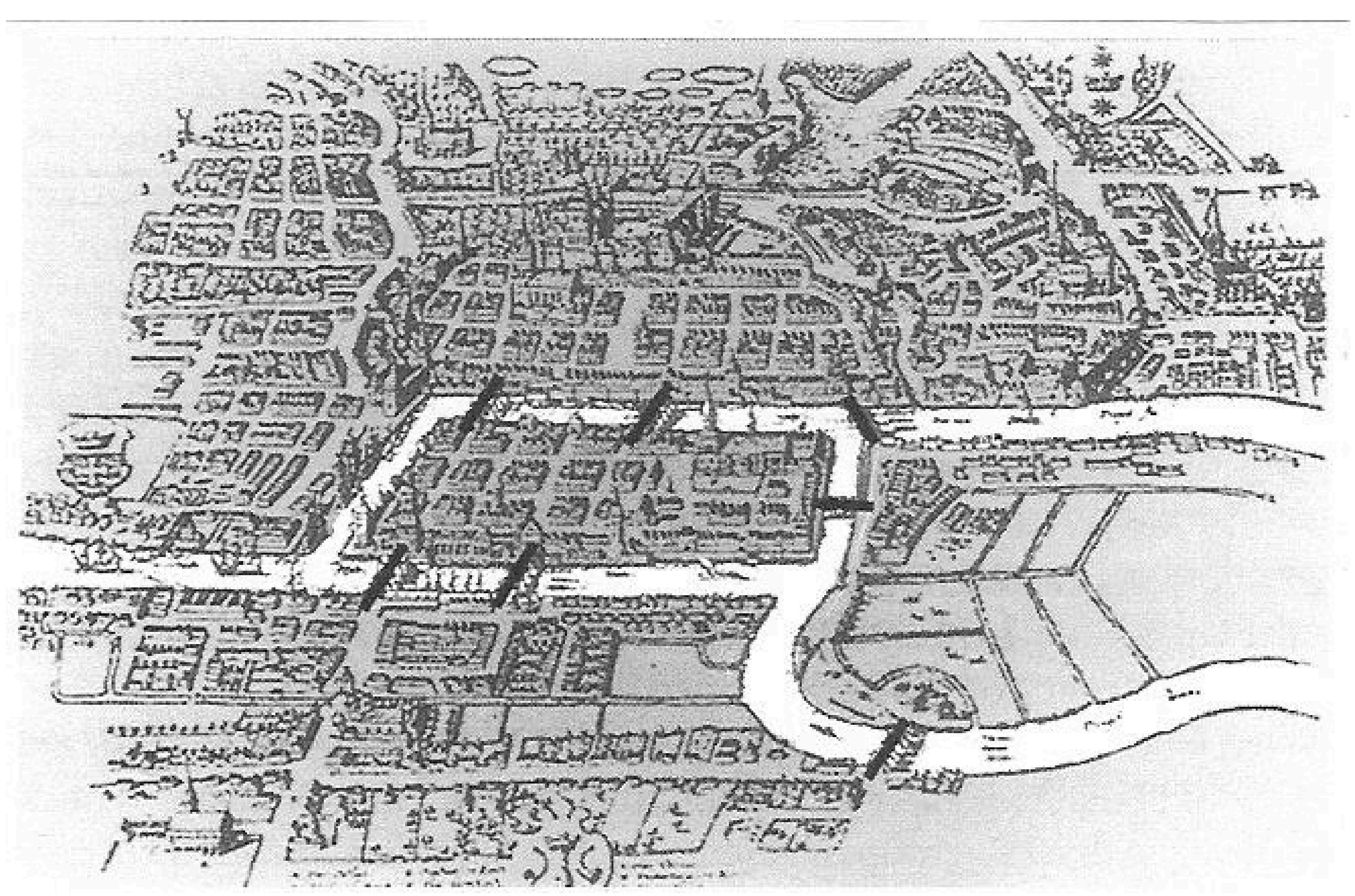


Graphs in math's are used for multiple logical and combinatoric problems. Helping solve complex problems with a simple visualisation and usually an exhaustive counting of the paths that meet a requirement or spotting the repeating subgraphs, graphs within a graph, and understanding the reason behind it.

Origins

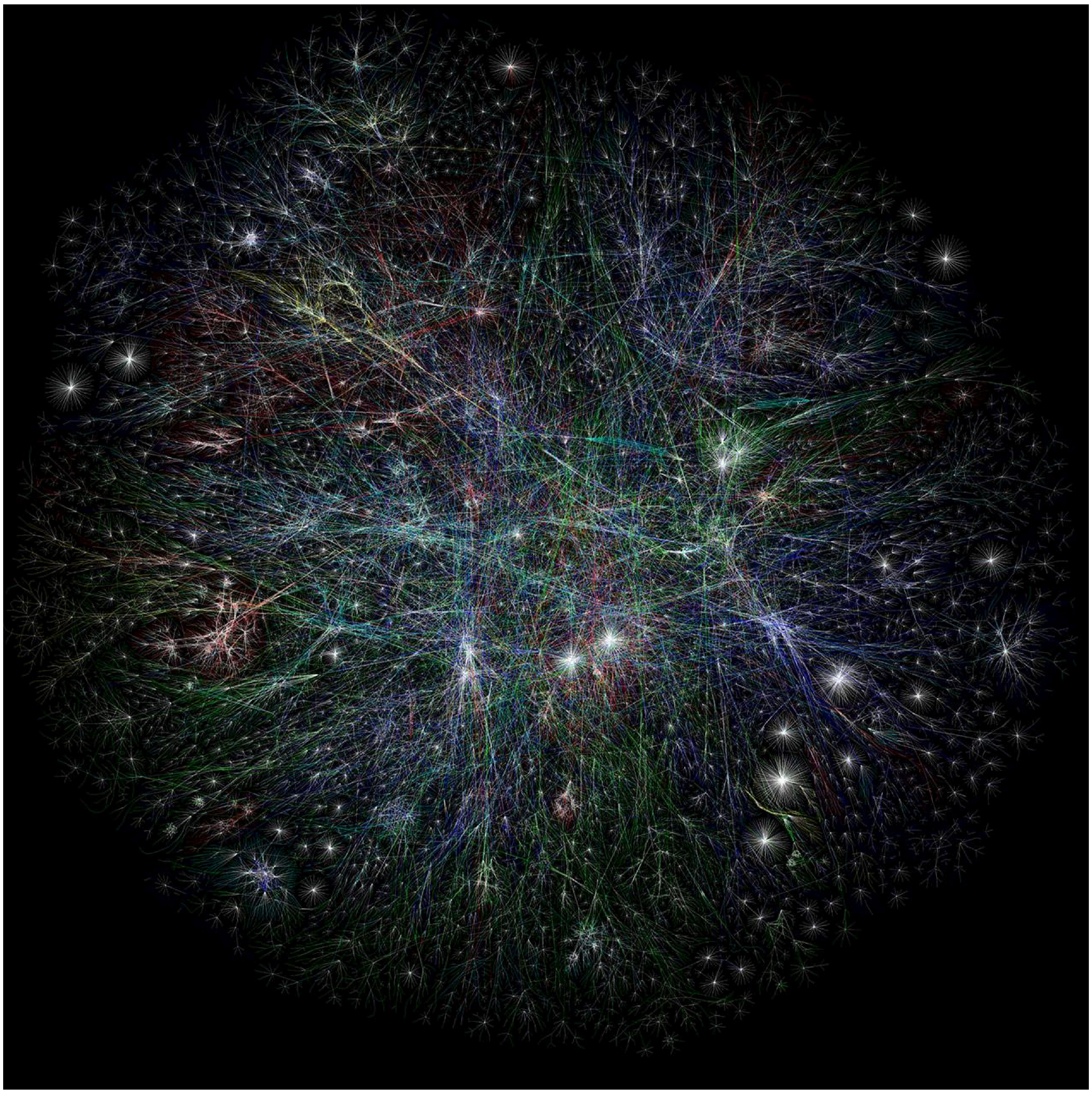
Graph theory was developed in the early 18th century when the famous mathematical problem of the seven bridges of Königsberg was presented. This involved the Prussian town of Königsberg, now Kaliningrad, Russia, which had seven bridges connecting 4 pieces of land across the River Pregel. The problem questioned whether giving a tour of the town while crossing each bridge exactly once was possible. With trial-and-error people found this problem to be very challenging, however in 1735, Leonhard Euler mathematically proved that this was impossible as each vertex had an odd number of connections.





Real life applications

At first graphs were used to show connections between cites or relationships between people, having quickly developed directed and weighted edges. Graphs then later in the 20th century, were used to show the current in an electric circuit allowing people to understand and develop electrical devices quicker and with less confusion. And recently AI neural networks and mapping programs have used graphs as a concept in their coding. People have also depicted the internet as a graph with communities and celebrities as central connections. This has also led to the idea of weighted nodes possibly being added to this expansive discipline.



Sources

Maths in Minutes by Paul Glendinning

Tingting H

The Birthday paradox

Sharing a birthday with someone. Few people actually know someone that they share a birthday with, personally, that is (searching it up online doesn't count!). But, what if I told you that the odds of someone sharing a birthday with somebody else are actually more likely than you think?

Let's say that we have a room of 23 people. What would you say the odds would be for two people to have the same birthday? 1/365 perhaps? Or maybe a bit less? What about just above half?

That's right, the answer is, in fact, just above half. Sounds weird, huh? But, due to probability, some events can be more likely to occur than we initially believe it to. The thing with this question is that it's asking for the odds of **two random people** sharing a birthday, not the probability that a specific person will share a birthday with another in the group. With this piece of information noted, it is much easier to find two people with the same birthday.

Many believe that the chances are 183, which is 365 days in a year divided by 2. This is because they are assuming the probability of a match increases linearly, when it actually increases exponentially (more and more) with every new person instead.

This problem is known as a veridical paradox, which usually results in an answer that is incomprehensible and absurd, but is true anyway. This means we can be fooled by our own intuition. The idea of 2 people out of 23 people having around a 50% chance of being able to share a birthday sounds like an incredibly small amount of people needed, hence making this a veridical paradox.

But, if you think about it, if we compare each individual's birthday to another individual's birthday, we make in total (23 x 22)/2 = 253 comparisons, which is far more than half the number of days in a year, making the 23 people needed seem far more reasonable than it initially seemed like.

And that's all there is to the Birthday Paradox, I hope not as confusing as it initially seemed after being broken down!

Yasmin P

Mersenne Primes

What is a Mersenne

A Mersenne Prime is a prime number which satisfies the formula (2^n)-1. In other words, it is a prime number that is one less than a power* of 2. Mersenne primes are named after a French polymath, Marin Mersenne who studied them in the early 1600s.

* The power also must be a prime number.

The first few Mersenne prime numbers are 3, 7, 31, 127, 8191...

And the exponents (n) that give these Mersenne primes are 2, 3, 5, 7, 13...

Mersenne's Marvelous Properties!

Mersenne primes have a close connection to **perfect numbers** (positive integers that are equal to the sum of its positive divisors**.)

** For example, 6 is a perfect number, as its factors are 1, 2 and 3, and the sum of those numbers is also 6.

How many Mersenne Primes are there?

It is conjectured (but not proven) that there are an infinite number of Mersenne primes, however, only 51 are known as of now.
Fun fact – the largest known prime number is **also** a Mersenne prime! It
is282,589,933– 1. This number is **24,862,048** digits long! In fact, all 6 of the **largest** prime numbers are Mersenne primes as well!

2	**3**	**5**	**7**	11	**13**	**17**	**19**
23	29	**31**	37	41	43	47	53
59	**61**	**67**	71	73	79	83	**89**
97	101	103	**107**	109	113	**127**	131
137	139	149	151	157	163	167	173
179	181	191	193	197	199	211	223
227	229	233	239	241	251	**257**	263
269	271	277	281	283	293	307	311

The first 64 prime exponents with those corresponding to Mersenne primes shaded in cyan and in bold, and those thought to do so by Mersenne in red and bold.

The search for Mersenne P r imes

Contributions to searching for Mersenne Prime numbers have started since 1461. Thanks to modern technology, the process of finding Mersenne Primes is a lot easier. The last Mersenne prime was found in December 2018, by a computer!

However, before calculators were used the search of Mersenne Primes was extremely difficult. Many mathematicians at this time tried to discover new Mersenne Primes. Unfortunately for these mathematicians during this period there was a large gap between the exponents yielding Mersenne Primes. After 127 the next exponent was 521 - more than 4 times larger than the previous record!

Yasmin P

Numeral Systems

Base-10 – The numeral system we use

You've probably learnt about Base-10inprimary school when you used Base-10 blocks. As a recap, Base-10 isthe way we assign **place value** to real numbers. It's also known as the ***decimal system*.***In Base-10, each digit of a number can have an integer value ranging from 0 to 9 (10 possibilities). The positions of the numbers are based on **powers of 10.** Each number position is 10 times the value to the right of it, hence the term Base-10.

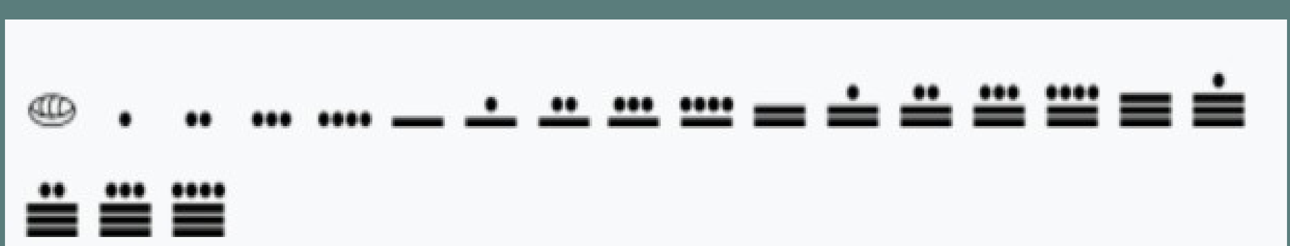
* This is because a digit's value in a number isdetermined by where it lies in relationto the decimal point (tens, hundreds,thousands, etc.)

Other Numeral Systems

The Babyloniannumeral systemused both Base-10 andBase-60. In fact, we still use Base 60 to measure quite a few things today! Examples include the measurement of time – 60 seconds, 60 minutes, etc. We also use Base-60 when measuring angles (like how we divide a circle into 360°).

Other base systems used throughout history include: *vigesimal* (base-20) which is anumeral systemused by the Mayans and the *duo-decimal* (base-12) numeral system used by Ancient Egyptians.

Binary, which you may have learnt about in Computer Science, is another numeral system. It is Base-2, as it only consists of the numbers 0 and 1.



Maya numerals before the 15th century.

Why do we use Base-10?

The reason why we, and many other cultures use Base-10 is quite simple – 10 is the best for counting, as we have 10 fingers.Counting using fingers has been a method of simplifying counting for thousands of years. In fact, did you know that the word "digit" is a synonym for "finger"? It comes from the Latin word *digit us.*

Where did Base-10 originate from?

Several civilisations developed theBase-10 system independently, including the Babylonians, the Chinese and the Aztecs. By the 7th century,Indian mathematicianshad perfected a Base-10system, which could represent any real number using only 10 unique sy m bol s.

What is the most **convenient** numeral syst em?

Many factors play into which numeral system is most convenient. Base-60 is very useful because it has manyfactors. However, Base-8 (octal)and Base-16 (hexadecimal) are a lot more useful when dealing with computers, as they are powers of two. Overall, the convenience of a numeral system depends on what it is being used for.

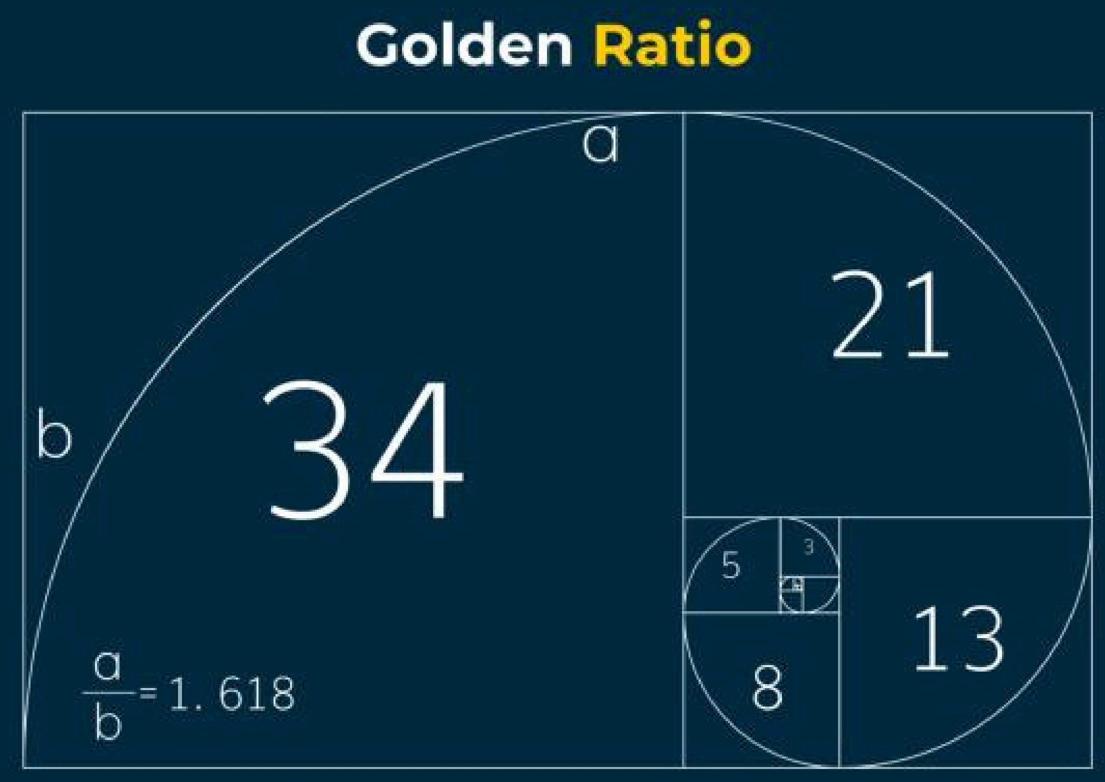
Yasmin P

The **Golden** Ratio

What is the Golden Ratio?

In the 13th century, an Italian mathematician introduced a sequence of numbers called the **Fibonacci sequence**. The rule this sequence follows is that the previous two numbers must be added together to find the next term. The first few terms are 1, 1, 2, 3, 5, 8, 13, 21...

In this sequence, the ratio between a term and its previous term gets closer and closer to the golden ratio, as you progress further. This ratio is approximately**1.618034**, or (1+√5)/2. It is denoted as the Greek letter**ϕ**, pronounced *phi*.



n	Fibonacci Terms	Ratio
1	1	
2	1	1
3	2	2
4	3	1.5
5	5	1.666666667
6	8	1.6
7	13	1.625
8	21	1.615384615
9	34	1.619047619
10	55	1.617647059
11	89	1.618181818
12	144	1.617977528
13	233	1.618055556
14	377	1.618025751
15	610	1.618037135
16	987	1.618032787
17	1597	1.618034448
18	2584	1.618033813
19	4181	1.618034056
20	6765	1.618033963
21	10946	1.618033999
22	17711	1.618033985
23	28657	1.61803399
24	46368	1.618033988
25	75025	1.618033989

What makes this ratio **special**?

There are many different reasons as to why this specific ratio is quite important, and nice – one way it's special is its appearances in **nature**. Phyllotaxis, the botanical name for leaf arrangement, is very closely linked to the Fibonacci sequence and the Golden Ratio. For example – in the florets of a large sunflower's head, there are two systems of spirals*radiating from the center. When you count carefully, there are **55** clockwise spirals and **34** counterclockwise spirals. Do these numbers seem familiar? They are **consecutive** terms in the Fibonacci sequence, and the ratio between these numbers is the **golden ratio**.

* You can find thesespirals in other plants as well, such ascauliflowers, certain types of cacti,and evenpinecones!

Formulas of the Golden Ratio

As previously mentioned,(1+√5)/2 is a formula for finding the Golden Ratio. Others includeϕ =1+1/(1+ 1/(1+ 1/(1+⋯ , ϕ or = 1 + (1/ϕ) , or evenϕ =2 × sin (54°). The golden ratio has many appearances in different areas ofmaths, as well as na ture!

Sources

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BLAISE PASCAL

Yasin A

Blaise Pascal

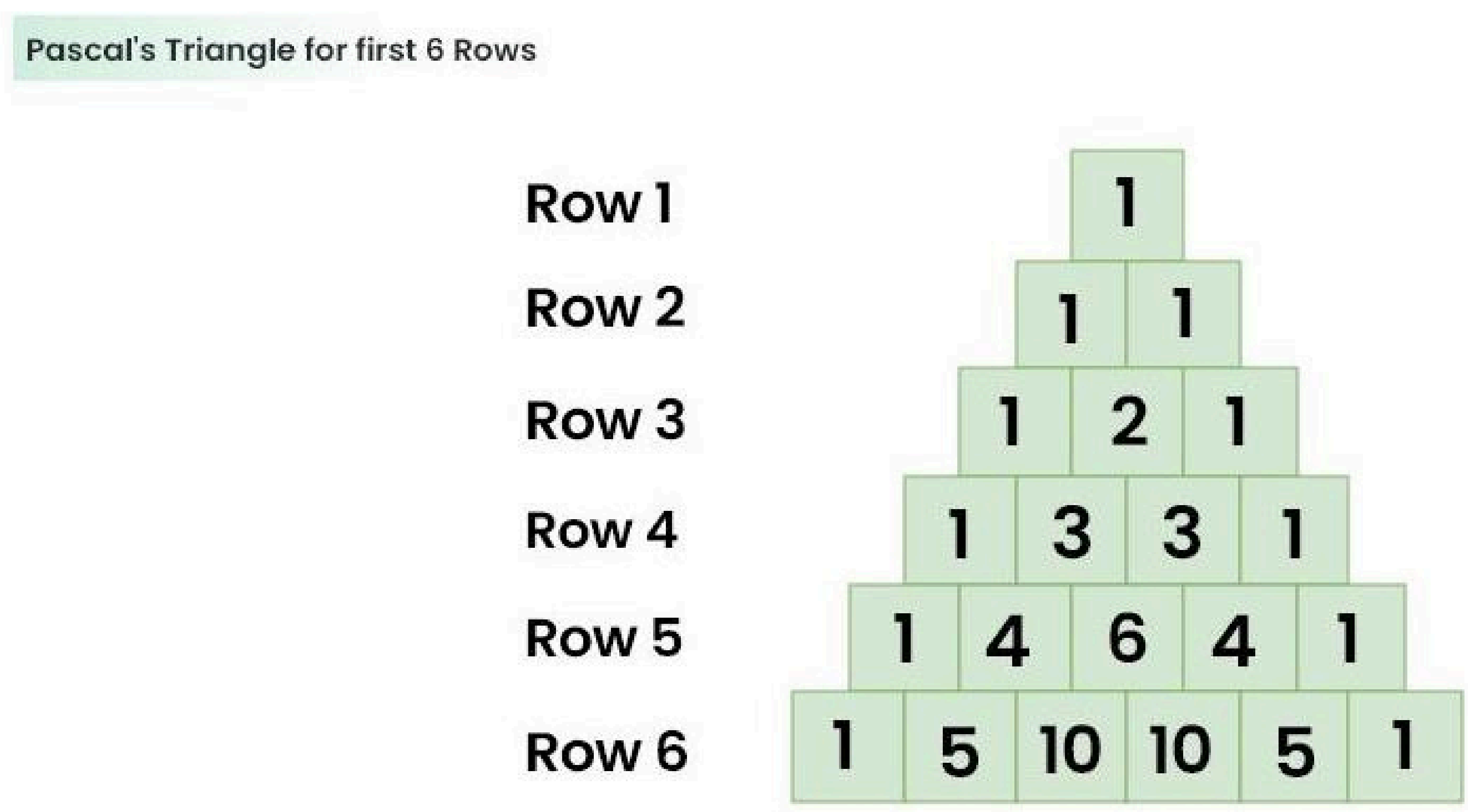
Blaise Pascal was born on June 19, 1623 in Clermont-Ferrand, France and died August 19, 1662 of stomach cancer. Pascal was a man of immense knowledge in addition to being a superb mathematician. Throughout his life, he developed into a scientist, inventor, philosopher and mathematician. Pascal made numerous Contributions to the Creation of the world at large including the hydraulic press, the barometer and the syringe he was revered as a God in the field of Mathematics and is better remembered for developing Pascal's Triangle and the adding machine which was originally known as the Pascaline and is today commonly recognised as a calculator.

Pascal’s Triangle

In 1653 he wrote the Treatise on the Arithmetical Triangle which today is known as Pascal’s Triangle. Although other mathematicians in Persia and China had independently discovered the triangle in the eleventh century, most of the properties and applications of the triangle were discovered by Pascal.

This triangle was among many of Pascal’s contributions to mathematics. He also came up with significant theorems in geometry, discovered the foundations of probability and calculus and also invented the Pascaline-calculator. Still, he is best known for his contributions to the Pascal triangle.

The easiest way to construct the triangle is to start at row zero and write only the number one. From there, to obtain the numbers in the following rows, add the number directly above and to the left of the number with the number above and to the right of it. If there are no numbers on the left or right side, replace a zero for that missing number and proceed with the addition. Here is an illustration of rows zero to six.



From the above figure, if we see diagonally, the first diagonal line is the list of ones, the second line is the list of counting numbers, the third diagonal is the list of triangular numbers and so on. Pascal's triangle formula is (n+1)C(r) = (n)C(r - 1) + (n)C(r). It means that the number of ways to choose r items out of a total of n + 1 items is the same as adding the number of ways to choose r - 1 items out of a total of n items and the number of ways to choose r items out of a total of n items.

SUDOKU

Estelle R

For classical Sudoku, the number of filled grids is 6,670,903,752,021,072,936,960, which reduces to 5,472,730,538 essentially different solutions under the validity preserving transformations. There are 26 possible types of symmetry, but they can only be found in about 0.005% of all filled grids. An ordinary puzzle with a unique solution must have at least 17 clues. There is a solvable puzzle with at most 21 clues for every solved grid. The largest minimal puzzle found so far has 40 clues in the 81 cells.

Many Sudokus have been found with 17 clues, although finding them is not a trivial task. A 2014 paper by Gary McGuire, Bastian Tugemann, and Gilles Civario proved that the minimum number of clues in any proper Sudoku is 17 through an exhaustive computational proof based on hitting set enumeration.

The problem of designing a system of clues that has a given grid of numbers as its unique solution can be formulated as a minimal hitting set problem. The 81 candidate clues from the given grid are the elements to be selected in the hitting set, and the sets to be hit are the sets of candidate clues that can eliminate each alternative solution. Therefore, the enumeration of minimal hitting sets can be used to find all systems of clues that have a given solution.

								1
							2	3
		4			5			
			1					
				3		6		
		7				5	8	
				6	7			
	1				4			
5	2							

17 clues and diagonal symmetry

	9			1			3	
		6		2		7		
			3		4			
2	1						9	8
		2	5		6	4		
	8						1	

18 clues and orthogonal symmetry

			9		2			
	4						5	
		2				3		
2								7
			4	5	6			
6								9
		7				8		
	3						4	
			2		7			

19 clues and two-way orthogonal symmetry

The fewest clues in a Sudoku with two-way diagonal symmetry (a 180° rotational symmetry) is believed to be 18, and in at least one case such a Sudoku also exhibits automorphism (an automorphism is a morphism of the object to itself that has an inverse morphism). A Sudoku with 24 clues, (a 90° rotational symmetry, which also includes a symmetry on both orthogonal axis, 180° rotational symmetry, and diagonal symmetry) is known to exist, but it is not known if this number of clues is minimal for this class of Sudoku

The number of minimal Sudokus (Sudokus in which no clue can be deleted without losing the uniqueness of the solution) is not precisely known. However, statistical techniques combined with a generator, show that there are approximately (with 0.065% relative error):

- 3.10×1037 distinct minimal puzzles
- 2.55×1025 minimal puzzles that are not pseudo-equivalent(same arrangement where all instances of one digit are switched with another digit).

A Sudoku solution grid is also a Latin square. There are significantly fewer Sudoku grids than Latin squares because Sudoku imposes additional regional constraints.

As in the case of Latin squares the (addition- or) multiplication tables of finite groups can be used to construct Sudokus and related tables of numbers. Namely, one has to take subgroups and quotient groups into account.

For the enumeration of all possible solutions, two solutions are considered distinct if any of their corresponding cell values differ. Symmetry relations between similar solutions are ignored (the rotations of a solution are considered distinct). Symmetries play a significant role in the enumeration strategy, but not in the count of all possible solutions.

In a 2005 study, the permutations of the top band used in valid solutions were analysed. Once the Band1 symmetries and equivalence classes for the partial grid solutions were identified, the completions of the lower two bands were constructed and counted for each equivalence class. Summing completions over the equivalence classes, weighted by class size, gives the total number of solutions as 6,670,903,752,021,072,936,960, confirming the value obtained.

Anoushka K

THE FIBONACCI SEQUENCE:

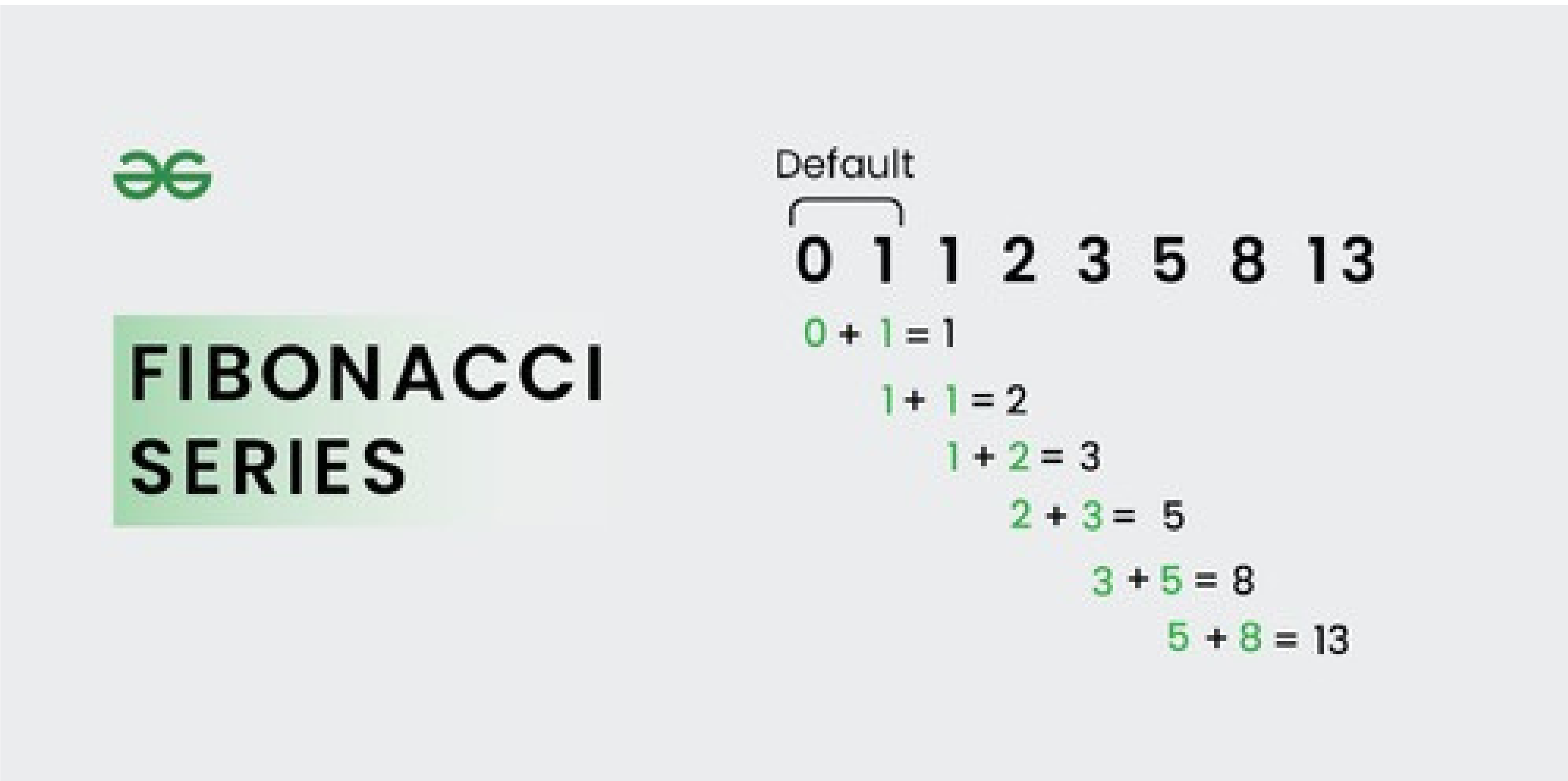
WHO WAS HE:

Fibonacci, his full name being Leonardo Bonacci, was an Italian mathematician born in around 1170 – (1240–50), who was later regarded as "the most talented Western mathematician of the Middle Ages". He was also known as "Leonardo the traveller of Pisa" through the plethora of different concepts he encountered through his travels.

It was through these travels where Fibonacci discovered the Hindu-Arabic numericals; these included the use of the digits 0–9 which is the main numerical system used in the world today. It was Fibonacci who brought this concept over to Europe, as he recognised the superiority of this system compared to the current previous numerical system, this led to the eradication of the use of Roman numericals. This new system allowed for a much more practical use of mathematical techniques: such as algebra, and the conversion of weights and measures.

However, Fibonacci was most known for his development of the **FIBONACCI SEQUENCE...**

Every number in the Fibonacci sequence is equal to the sum of the two numbers that came before it. Although some versions begin with 1 and 1, the sequence usually begins with 0 and 1. The order is as follows:



This sequence is ultimately an infinite sequence and so is mathematically defined by the formula:

$$F(n)=F(n-1)+F(n-2)$$

with initial conditions:

$$F(0)=0$$

$$F(1)=1$$

USES OF THIS SEQUENCE:

Numerous natural occurrences exhibit the Fibonacci sequence. For example, if you were to look at the spiral of seeds in the centre of a sunflower and count them, your total will be a Fibonacci number. This also works for the number of petals of some flowers eg: daisies have 13 petals, lily's have 5. Because of its qualities and connections to the golden ratio, it is also utilised in computer algorithms, financial markets (in technical analysis), and numerous other fields of science and art. For instance, Fibonacci retracements are popular amongst traders to predict potential future prices in financial markets. This is because they can help to identify potential support and resistance levels where the price of assets may find a price floor/ ceiling after a significant move up or down.

Micah E

Exploring Modular Arithmetic

Modular arithmetic involves performing arithmetic operations on remainders, where numbers reset to zero after reaching a specified value called the modulus. This cyclic nature makes modular arithmetic essential in fields like cryptography, computer science, and number theory.

Introduction to Modular Arithmetic

Modular arithmetic centres on remainders and cycles. Numbers "wrap around" after reaching a modulus, such as in a system where numbers reset after 5. The modulo operator (%) calculates remainders and congruence relation (≡) shows that two numbers have the same remainder for a given modulus. For example, $15 \equiv 3 \pmod{12}$ means 15 and 3 share the same remainder when divided by 12. This concept is seen in a 12-hour clock: if it's 7:00 and we add 8 hours, we get 15:00, but on the clock, the time is 3:00, illustrating the cyclic nature of modular arithmetic.

Properties of Modular Arithmetic

Modular arithmetic shares properties with standard arithmetic. It is closed under addition and multiplication, meaning the sum and product of two integers modulo m result in another integer modulo m. It also maintains commutativity and associativity and is distributive, allowing multiplication to distribute over addition. These properties allow us to manipulate modular equations and solve congruences.

Applications in Cryptography

Cryptography heavily relies on modular arithmetic. The RSA algorithm, for example, uses modular exponentiation to generate public and private keys for encryption and decryption. Large prime numbers are chosen to compute the modulus and derive keys. Messages are encrypted with the public key and decrypted with the private key. The security of RSA encryption depends on the difficulty of factoring large composite numbers and computing modular inverses, ensuring the confidentiality and integrity of data.

Applications in Computer Science

Beyond cryptography, modular arithmetic is crucial in generating pseudo-random numbers, which simulate randomness for simulations, statistical sampling, and gaming. These numbers are vital in weather forecasting, unbiased statistical analysis, and creating unpredictable gaming experiences. Pseudo-random numbers, though deterministically generated, effectively mimic true randomness, making them invaluable in various computer science applications.

Modular arithmetic combines theoretical depth with practical versatility, profoundly impacting mathematics and real-world problems. Its applications span from ancient civilizations to modern cryptography and computer science, highlighting its enduring relevance. Further exploration into this fascinating subject reveals its potential to solve complex challenges in the digital age.

Vidhi M

The Role of Maths in Stock Market Investments

Investing involves the practical application of various mathematical concepts. Although investing appears to be primarily influenced by market trends and economic indicators, it fundamentally relies on numerical analysis. Understanding the types of math that are most beneficial for investing can improve your decision-making process. This essay explores important mathematical concepts, including simple arithmetic, percentages, compounding, statistics, probability, and calculus, and how they apply to stock market investments.

Simple Arithmetic and Algebra in Stock Market Investments

At the most fundamental level, investing relies on simple arithmetic to calculate investment returns, profit margins, and dividend yields. Basic arithmetic is essential for all investment calculations, from earnings per share to the price-to-earnings ratio, enabling informed investment decisions. Several fundamental equations are also essential for investors to understand. These equations provide insights into the performance and potential of investments.

For example:

1 Return on Equity (ROE)

$$\text{ROE} = \frac{\text{Net income}}{\text{Shareholder equity}}$$

ROE, assesses how effectively a company utilises shareholders' funds to generate profits. It's calculated from financial statements, with a higher percentage suggesting better returns. Comparing to industry averages is crucial, as high ROE can be influenced by debt levels. Therefore, careful analysis is needed for investment decisions.

2 Future Value of Investment

$$F = P \times (1 + R)^t$$

Where:

F = Future value of the investment

P = Present value of the investment

t = Number of compounding periods

R = Periodic interest rate or rate of return

This formula helps investors estimate the future value of their investments, allowing them to plan how much they need to invest each year to achieve their financial goals.

3 Capital Asset Pricing Model (CAPM)

$$\text{Stock Price} = V + B \times M$$

Where:

- V = Stock's variance
- B = How the stock fluctuates with the market
- M = Market level

CAPM assesses the price of a stock in relation to general market movements, helping investors understand a stock's behaviour in different market conditions.

Percentages

- Vidhi M

They are a crucial concept in investing, used to represent various forms of measurement like investment returns, stock price changes, and company growth rates. Understanding how to calculate and interpret percentages enables comparison of investments and identification of trends. For instance, a 10% stock price increase might seem positive, but if the market rose by 20%, the stock underperformed.

Compounding

Compounding is one of the most powerful concepts in investing. It's the idea that you can earn returns not just on your original investment but also on the returns you've already earned.

- A is the amount of money accumulated after n years, including interest.
- P is the principal amount (the initial amount of money).
- r is the annual interest rate.
- n is the number of times that interest compounds per year.
- t is the length of time the money is invested (in years).

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

This principle highlights the benefits of long-term investing and reinvesting returns, demonstrating how small investments can grow significantly over time. Therefore, starting to invest early is beneficial.

Statistics: Understanding Data

Key statistical concepts in investing include mean, median, mode, range, standard deviation, and correlation.

For instance, the mean return of a stock over a specific period provides an idea of its average performance, while the standard deviation indicates how much the stock's returns deviate from the mean, reflecting its risk level. Correlation shows how closely the performances of two stocks are linked.

Statistics help identify market trends and patterns; for example, regression analysis can be used to assess how factors like earnings, interest rates, or economic indicators influence a stock's price.

Probability

As an investor, you can use probability to evaluate investment risks and predict future performance. For instance, estimating the probability of a startup's success can be based on data from similar startups. It also helps assess the likelihood of market scenarios, such as estimating the chance of a recession, allowing you to adjust your investment strategy accordingly

Calculus: Understanding Change Over Time

Calculus, precisely differential calculus, can be helpful in investing because it deals with rates of change. Surroundings are constantly changing in finance, and understanding these changes can be key to making good investment decisions. You can utilize calculus to determine the growth rate of a company's earnings or to estimate the rate at which interest rates are likely to change. More advanced investment strategies, like options pricing, also use calculus.

In conclusion, mastering these mathematical concepts helps investors make smart choices and these skills are incredibly helpful for successful investing.

By- Vidhi Malik 12P

Game Theory

Dylan L

Game theory is often defined as the science of strategy. It analyses the behaviour of 2 or more participants in situations involving gains or losses. Not only do games such as chess and poker use it, but seemingly competing companies and businesses do too.

There are many underlying strategies and theories that compile to create the umbrella term, "Game Theory" and I will be talking about the application of game theory in football using a "mixed strategy". In penalties both the striker and goalkeeper have 3 choices: left, right and centre. If we look at past figures, out of 100 000 goals, 75.5% of them were successful, 17.6% were saved by goalkeepers and the rest were either wide or hit the post. Majority of shots taken were either left or right rather than the centre.

We must note that strikers will rarely always go the same way for every penalty they take as randomness is needed so the goalkeeper doesn't always guess your direction correctly and saves it this. So we will used a mixed strategy. Below is a graph which shows the probabilties of a striker scoring given the goalkeeper goes that way and the striker goes their respective way.

	Keeper to (Kicker's) Right	Keeper Centre	Keeper Left
Kicker aims Right	0.5	0.8	0.9
Kicker aims Centre	0.8	0.3	0.8
Kicker aims Left	0.9	0.8	0.5

The expected average score will help in finding out which is the best combination to use. One case can be if both players decide a direction randomly, so probability 1/3. Calculating the expected value would be (1/9 x 0.5) + (1/9 x 0.8) +...+ (1/9x0.5) = 0.7. Now if the striker instead chooses to go left half their shots and right for the other half the expected score changes as 1/3 becomes 1/2. This equals 0.73. We can see 0.73>0.7 so the striker should use this strategy. However, the goalkeeper will pick up this pattern which in the long term may result in less goals scored. Taking into account various combinations and using a technique called linear programming we can find the best combination for the striker and then for the keeper. For the striker the values are 42% right, 42% left and 16% central. The expected value for this is 0.72 which we can see is roughly similar to 0.75 as past data demonstrates.

Game theory provides insight into behaviour. So it is important to remember that it doesn't tell us directly what the aim should be but rather how someone can best achieve their aim.

Maths@Cambridge Residential

Rose E

I recently went on a maths residential to Cambridge, where I was surrounded by girls my age, from all around the UK from Ireland to Dover, interested in pursuing maths as a degree at competitive universities. As we arrived at Cambridge, the first thing that struck me was the number of bikes on the road. Cambridge is substantially more bike-friendly than London, and it was nice to see less congestion due to cars, vans and trucks on the roads.

After putting our bags away and getting used to our accommodation (We rooms all to ourselves with ensuite bathrooms!), we then gathered into a lecture room, and got to know the people around us, and the undergraduate students who would be helping us through our residential, and answering our questions. There were a range of undergraduates from many different subjects all related to maths, like physics, engineering and economics.

We then had dinner in the dining hall of Christ's college. It was very fancy, and they gave a whole Hogwarts experience, giving us an opportunity to wear formal gowns too. We learnt many interesting facts about Christ's, like how it was founded by a woman, Lady Margaret Beaufort, but she was not allowed to enter Christ's, so she made a secret tunnel so that she could see how the construction work was going. We were also able to speak to many undergraduates and academics, to ask them about their opinions on Cambridge, and the overall experience they have had of the city. It was very enlightening, and it gave me a lot more insight into what to consider when I do apply to universities.

On Friday, we had breakfast in the Upper Hall in Christ's college. We then had an hour and 45 minutes of free time, which I spent with a group of people walking around Cambridge and exploring all the nearby colleges. As we were prospective student, we were allowed to enter all the colleges free of charge, including King's college, which was quite exciting. We then went to Trinity College, and had a tour of the grounds. In the library, they showed us how they had many first editions of rare books, like the first manuscripts for Winnie the Pooh, and the original versions of Shakespeare's plays. We managed to see some of the work of an extremely important Indian mathematician, Srinivasa Ramanujan. It was all very complicated, and difficult to understand, but it was nice to see some diversity in the history of maths.

There was also Wittgenstein's notebooks. Wittgenstein was one of the most influential figures in 20th century Philosophy. They also had one of the very first edition of Philosophiae Naturalis Principia Mathematica, one of the first books published by Isaac Newton, with some of his very own annotations! Some other things of Newton's that were on display included a lock of his hair, his prism, his writing instruments, and his undergraduate notebook.

We had lunch in the Trinity Great hall, and then we had a lecture about admissions into Cambridge, which was very insightful, and gave us lots of tips on how to have the strongest application, but overall, the main thing was that you had to be passionate about maths, and have talent in it. We then had a lecture about a maths adjacent subject. The options we had were land economy, economics, bioengineering and physics. I chose physics, and the lecture was mainly about fluorescence. It was quite difficult to follow, especially with the many long equations, and university-level physics, but sometimes the lecturer would start talking about something I was learning in Physics at the moment, like the Photoelectric effect, wave-particle duality, and Young's double-slit experiment, and I found it so satisfying to see how easily the things we learn at A-Level can be linked to complicated things at university.

We had a student life Q&A with some undergraduate and PhD students at Cambridge, and we were able to see more about what it meant being a student at Cambridge. They spoke a lot to us about the collegiate system, how the university was separated into colleges, and the colleges were the ones who interviewed and selected the people who got offers. We also got to know about how effective and easy to use the wellbeing system at Cambridge is, and how if someone at Cambridge ever felt to overwhelmed with the workload, or anything, there are so many people they could go to for help, which is very reassuring.

We then had a lecture about how maths matters from Dr Robert Hunt, a Fellow and Director of Studies in Mathematics in Christ's college. He spoke to us about Paul Dirac, a mathematician who loved maths because of how beautiful certain things in maths can be.

Paul Dirac formed an equation for the electron, but it was an extremely difficult equation, with things like complex numbers and 4-dimensional vectors in it, so, using the power of mathematical notations, he managed to simplify it into a very small, beautiful equation, that still had the same complexity and accuracy of the first. When he published it, there were many values for which it worked, however, there were also some values for which if it were true, then the value for energy would have to be negative. When people tried to tell Dirac that negative energy could not be real, and that his equation was wrong, he refused to believe it. When asked why, he simply said that it was too beautiful to be wrong. Later, in August 1932, using Dirac's equation, Carl David Anderson discovered the positron, proving Dirac's equation as correct. It was amazing how maths can be used as a basis to discover extraordinary things that at first seem impossible.

Saturday was our last day, so we had to give in our key cards and get all of our bags. We had one final lecture about prime numbers from Dr Henry Bradford, Fellow and Director of studies in Mathematics at Christ's college, who talked to us about how they are used in encryption, and showed us three different methods on how to prove that there are infinite primes.

Overall, I believe this experience at Christ's has been very beneficial, as it really opened my eyes as to what university life at Cambridge really was like. I used to believe that getting into such competitive universities like Oxford and Cambridge was not something I was capable of, but after experiencing this residential, and getting the tips from admissions staff on how to make your application stand out, it is starting to seem more like something that is possible for me.

FOURIER SERIES

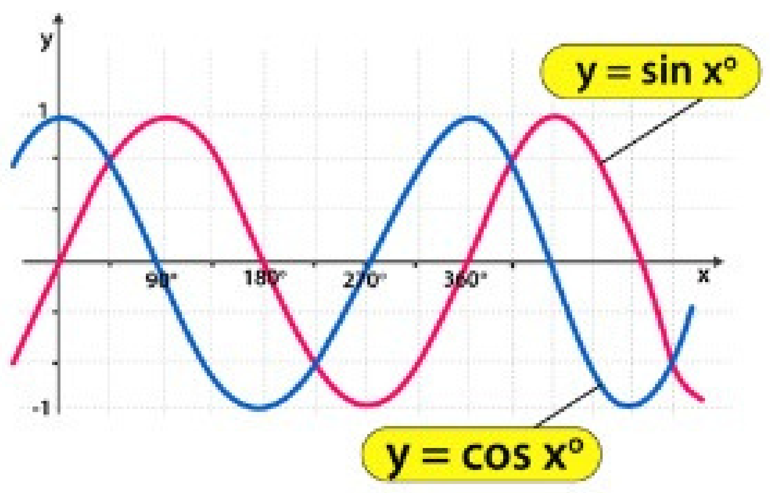
Glenda P

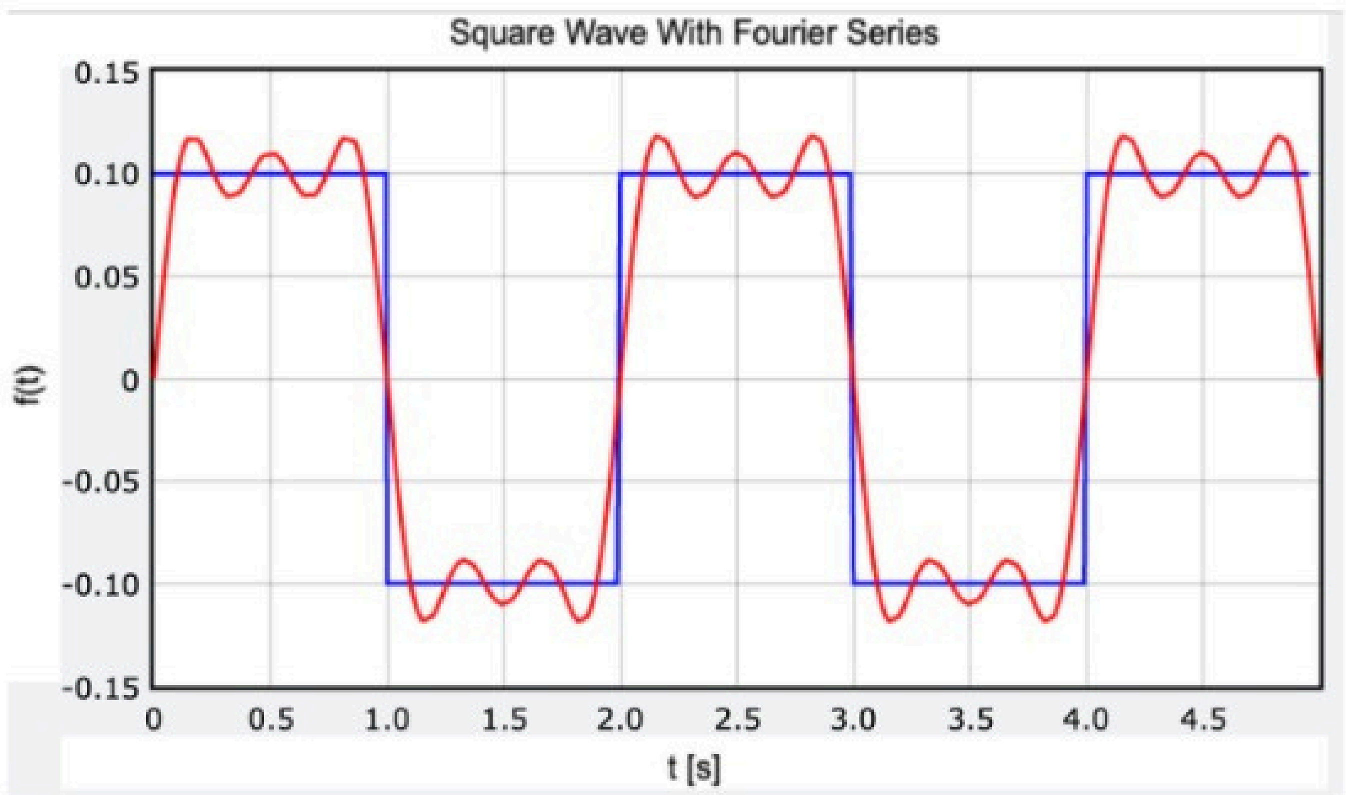
WHAT IS THE FOURIER SERIES?

The Fourier series is an expansion of a periodic function f(x) in terms of an infinite sum of sines and cosines. It was introduced by a French mathematician named Joseph Fourier, who was trying to study the flow of heat in a metal plate and decided to express the heat source as an infinite series of sine and cosine functions. This is because the temperature distribution can be described using several sine and cosine waves.

In Maths, you learn about the trigonometric functions sine and cosine and how they form waves on a graph. One of the key features of these waves is that they are periodic waves. A periodic wave is a wave that repeats after a certain length of time and for sine and cosine waves, they repeat every 360 ° / 2π. If you take any position on the sine or cosine graph and move it over by 2π, you will get the exact same height. This can be expressed as a period function with "T" as the period of the wave.

$$f(t+T) = f(t)$$





There are other types of periodic waves such as square waves, which also has a period of 2π. It can be approximated using sin(x) as the peaks and troughs roughly align with the square wave at 1 and -1. To obtain a better approximation you need to add sine waves with a faster oscillation, but smaller amplitude. It will look something like the following picture.

THE FOURIER SERIES EQUATION:

$$f(x) = \frac{1}{2}a_0 + \Sigma_{n=1}^{\infty} a_n \cos nx + \Sigma_{n=1}^{\infty} b_n \sin nx$$

where,

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

n = 1,2,3 .....

f(x) – the function being transformed

a0, an and bn – Fourier coefficients and represents the amplitude

n – indexes the harmonics

Glenda P

APPLICATIONS OF THE FOURIER SERIES:

The Fourier series can be applied to a wide range of mathematical and physical problems.
Some examples of this include:

1) Selective filtering

With the help of the Fourier series filters can be designed to filter specific frequencies. This is particularly useful in filtering applications such as the radio where you would want to remove unwanted frequency components from a signal, whilst maintaining the desired frequency.

2) Speech recognition

Speech patterns can be processed and recognised with the Fourier series since the frequency of the sound can be detected.

3) Compression of Signals

Unnecessary information can be removed by the Fourier series allowing signals to be compressed. Along with the Fourier series for periodic signals, there is also the Fourier transform, which applies to non-periodic signals. They work together for image compression, removing high-frequency components the human eye cannot detect. A well-known example would be the JPEG image format.

4) Noise Filtering

In addition to selective filtering, the Fourier series can enable noise filtering. It can completely remove any unwanted noise from an audio signal. A great example of where this applies is noise-cancelling headphones.

Overall, the application of the Fourier series has limitless applications and is an essential concept in engineering and mathematics when working with waves.

Tosin A

Math and Music

The Harmonious Relationship Between Numbers and Notes

The connection between mathematics and music is profound and multifaceted, revealing the deep structural similarities between these two seemingly disparate fields. Both disciplines are governed by patterns, structures, and relationships that are as beautiful as they are complex. Understanding the mathematical foundations of music not only enhances our appreciation of musical compositions but also demonstrates the universality of mathematical principles.

Mathematical Foundations of Musical Scales

At the heart of musical theory is the concept of scales, which are sequences of notes arranged in ascending or descending order. The most common scale in Western music is the diatonic scale, which consists of seven notes. The frequencies of these notes are related by specific ratios. For example, the octave, which is the interval between one musical pitch and another with double its frequency, can be expressed as a 2:1 ratio. Similarly, other intervals like fifths (3:2) and fourths (4:3) have precise mathematical relationships. These ratios form the basis of harmony and consonance in music.

Rhythm and Time Signatures

Rhythm in music is inherently mathematical, involving the division of time into equal parts. Time signatures, such as 4/4 or 3/4, dictate how many beats are in each measure and what note value constitutes one beat. This division and subdivision of time can be described using fractions and sequences, providing a clear link between rhythmic patterns and mathematical concepts. Polyrhythms, where multiple rhythmic patterns are played simultaneously, showcase more complex mathematical relationships, creating intricate and captivating musical textures.

Harmony and Chord Progressions

Harmony, the simultaneous combination of notes to produce chords, is deeply rooted in mathematical principles. Chords are built from scales using specific intervals, and their progression follows patterns that can be analyzed mathematically. The Circle of Fifths, a visual representation of the relationships among the twelve tones of the chromatic scale, illustrates how chords are related and helps musicians understand key signatures and modulations. This cyclical pattern is a clear example of how mathematical structures underpin musical harmony.

Fourier Transform and Sound Analysis

The Fourier Transform, a mathematical technique that decomposes functions into their constituent frequencies, has significant applications in music. It allows us to analyze complex sounds by breaking them down into simpler sinusoidal components. This analysis is essential for understanding timbre, the quality of a musical note that distinguishes different instruments. By applying Fourier analysis, we can visualize and manipulate the frequency spectrum of sounds, leading to advancements in audio technology and music production.

Algorithmic Composition and Artificial Intelligence

In recent years, the intersection of mathematics and music has expanded into the realm of algorithmic composition and artificial intelligence. Composers and researchers use algorithms to generate music, employing mathematical models to create melodies, harmonies, and rhythms. AI systems can analyze existing musical pieces, learn their patterns, and produce original compositions, showcasing the creative potential of combining mathematics with music theory. Furthermore, AI can also give musicians ideas and inspiration for their next music album or sonng.

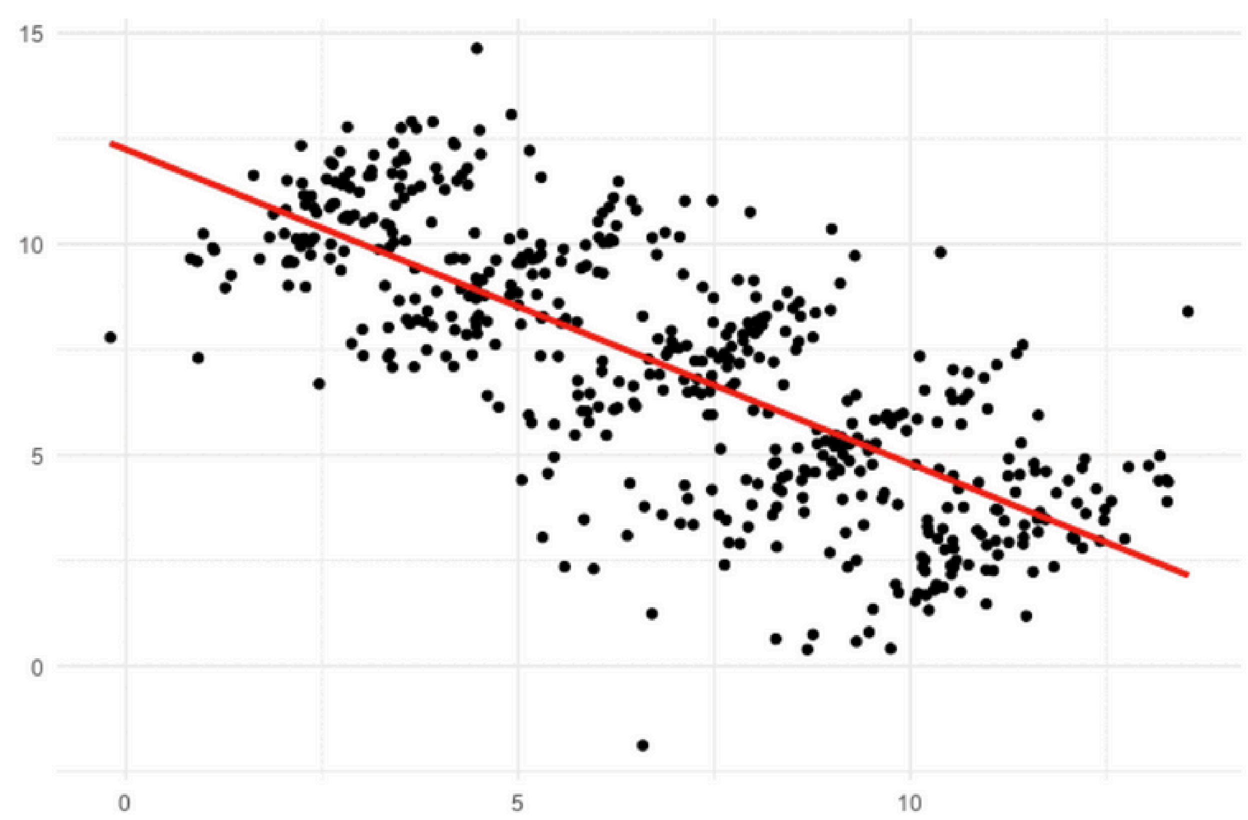
The relationship between mathematics and music is a testament to the universal language of patterns and structures. From the precise ratios that define musical scales to the complex algorithms that compose new music, mathematics provides a foundation for understanding and creating music. This harmonious relationship not only enriches our appreciation of both fields but also highlights the beauty of their unification. Whether you are a mathematician fascinated by music or a musician intrigued by numbers, exploring this relationship offers endless opportunities for discovery and creativity.

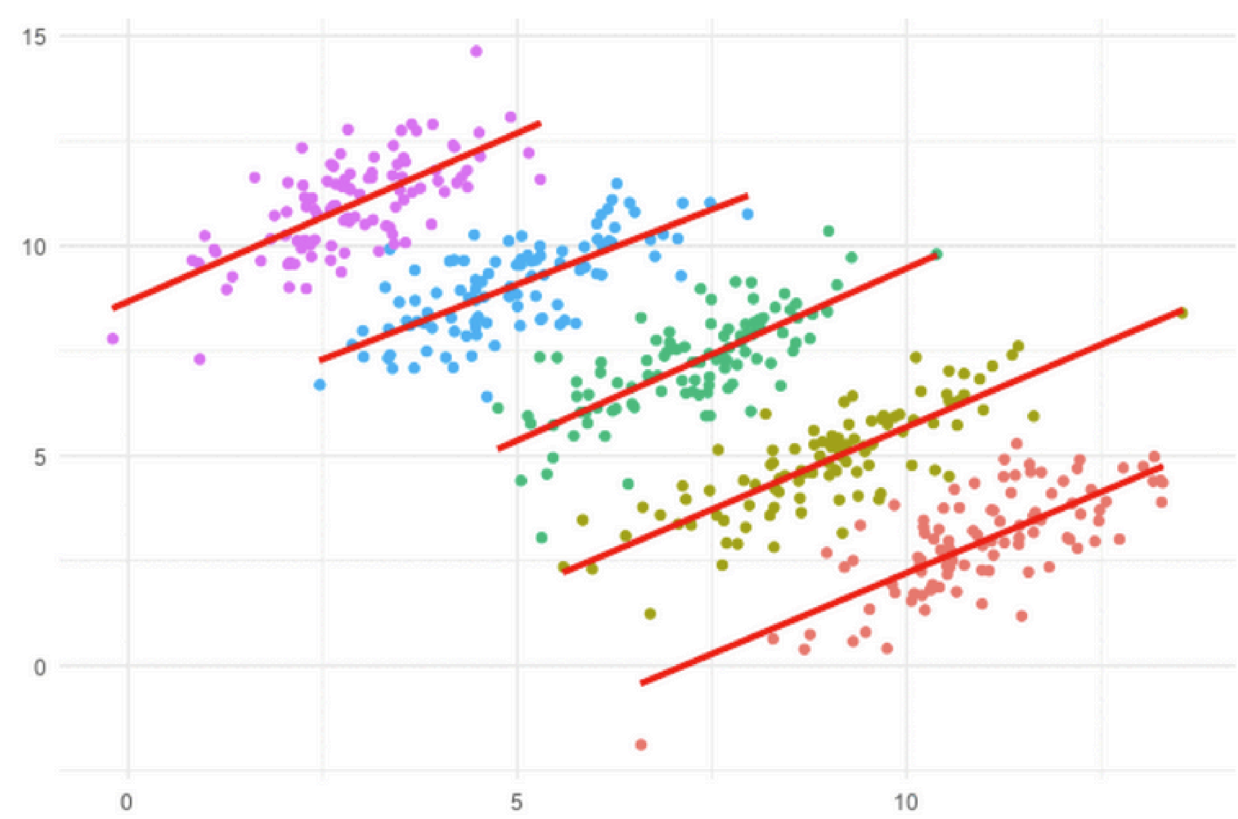
Simpson's Paradox

Joana T

We are used to deciding one single type of correlation in a dataset, but what if there were multiple? Simpson's Paradox is a phenomenon where an association between two variables in a population emerges, disappears or reverses when the population is divided into subpopulations.

It can happen quite often and is very easy to miss when analysing data and drawing conclusions. If it goes unnoticed, the conclusions drawn can be misleading and potentially incorrect. One possible reason for the paradox is a 3rd confounding* variable. What this means is that the presence of a 3rd variable is affecting the relationship between the 2 variables we are testing. Because the influence of the 3rd variable is not taken into account, Simpson's paradox therefore occurs.





The graphs above are a visualisation of Simpson's paradox. On the left, we can see the overall correlation (negative), however when the data is split into subgroups or strata, the correlation reverses. It becomes positive.

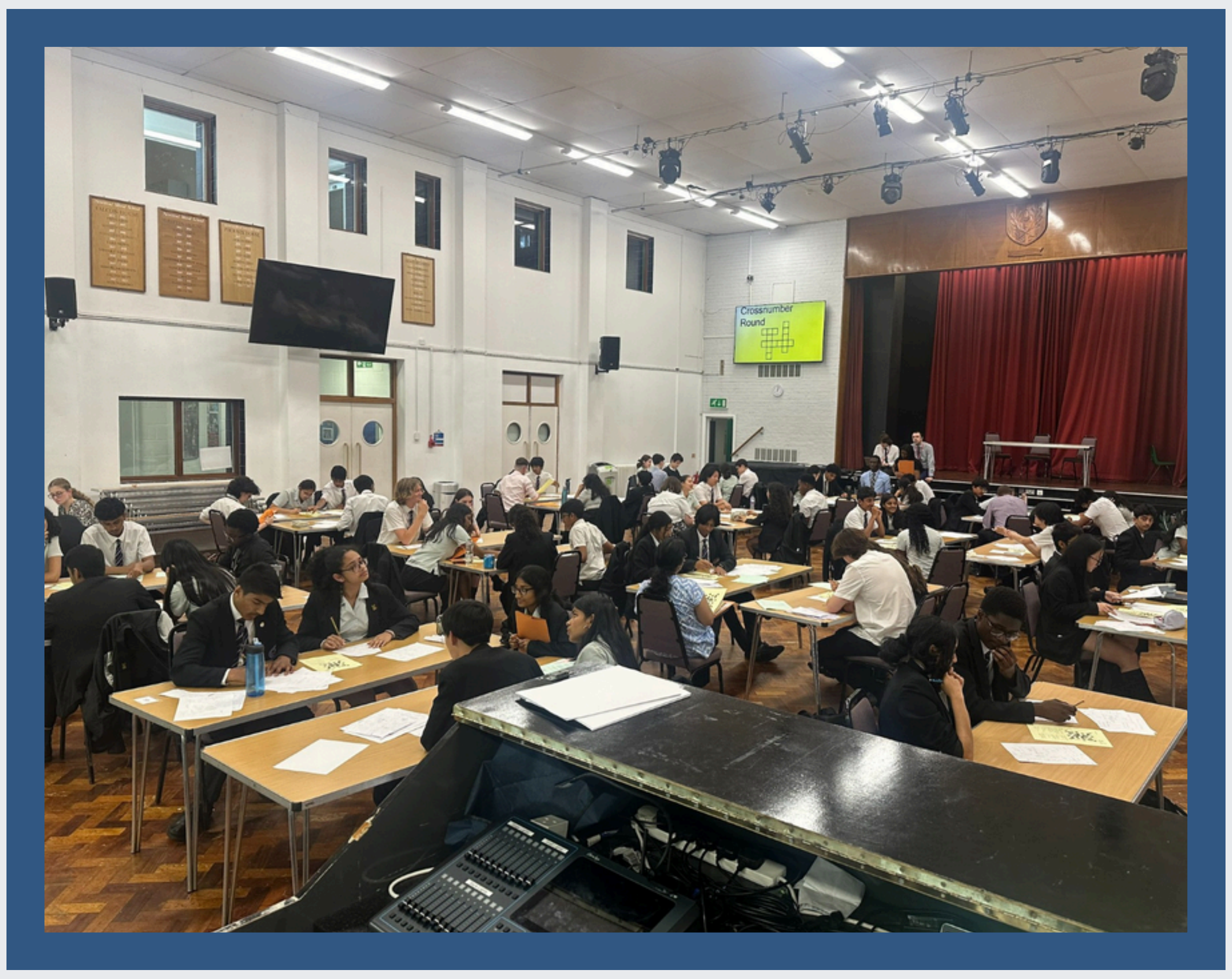
One of the most famous situations in which this paradox is found, is university/college admissions. At UC Berkeley, 44% of the men which applied, were admitted, and 35% of the women which applied were admitted. We would be immediately inclined to think there is a preference towards men. However, when we look at each department, we find that women tended to apply to courses with harsher admission rates than men. Here is a table of the largest 6 departments at UC Berkeley.

Department	Men	Women
1	62%	82%
2	63%	68%
3	37%	34%
4	33%	35%
5	28%	24%
6	6%	7%

We can see that on average, women were accepted at a roughly equal or higher rate than men but due to this 3rd confounding factor (concentration of women and men respective to difficulty of course), it creates this interesting effect of Simpson's paradox.

**confounding = an unmeasured variable that influences both the supposed cause and effect.*

newstead+olaves
maths comp

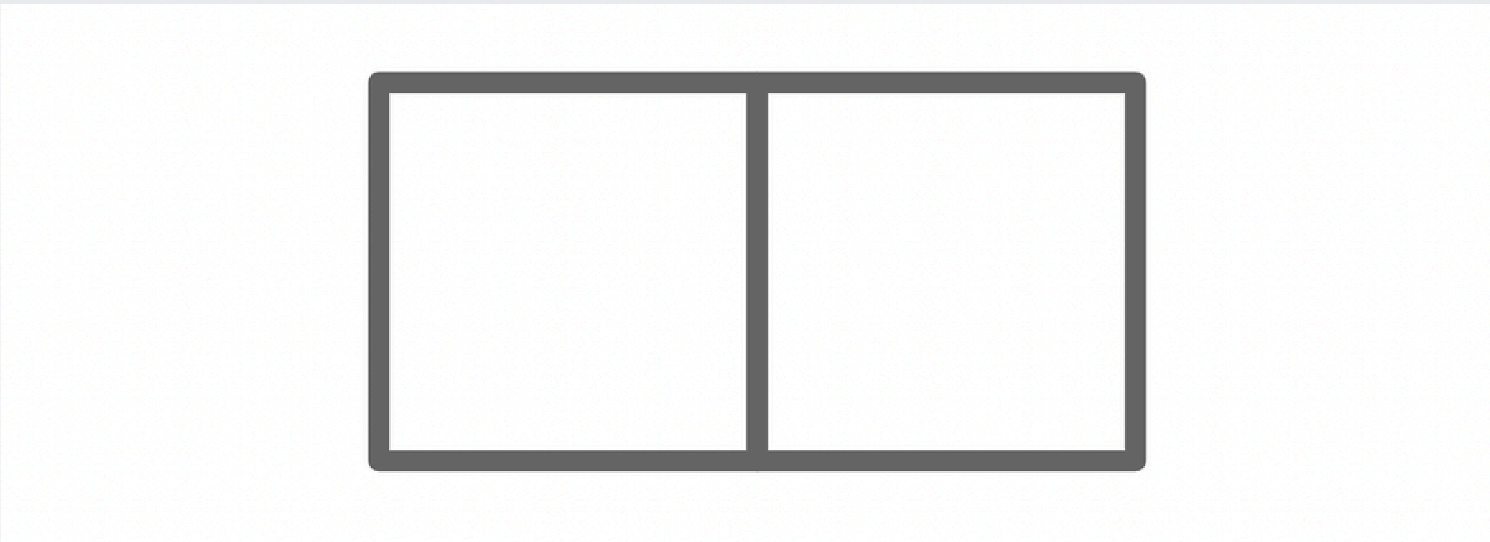


On the 25th June, Newstead hosted an inter-school maths competition, with 32 students from both Newstead, and St Olaves. Each team consisted of two Newstead students, and two Olaves students. After the initial warm up, there was a group round, where the team was allowed to work together to solve questions. The second round had a series of questions, each worth five marks, where the team with the closest answer won the points. In the third round, the teams split in half, with a Newstead student and an Olaves student on each side, where one side's answer would be given to the other side, to use as part of their solution. Twelve points were up for grabs, with three extra points available if questions were done fast enough. The fourth round was exactly like the second, and was followed by a cross number round, where again, the teams divided into two and both sides of the team would give each other clues with their answers. To wrap it up, the sixth round was formatted like the second and fourth rounds, and points were added up to determine the team with the highest amount of points overall.

-winning team, sophia w

Riddler QUESTIONS

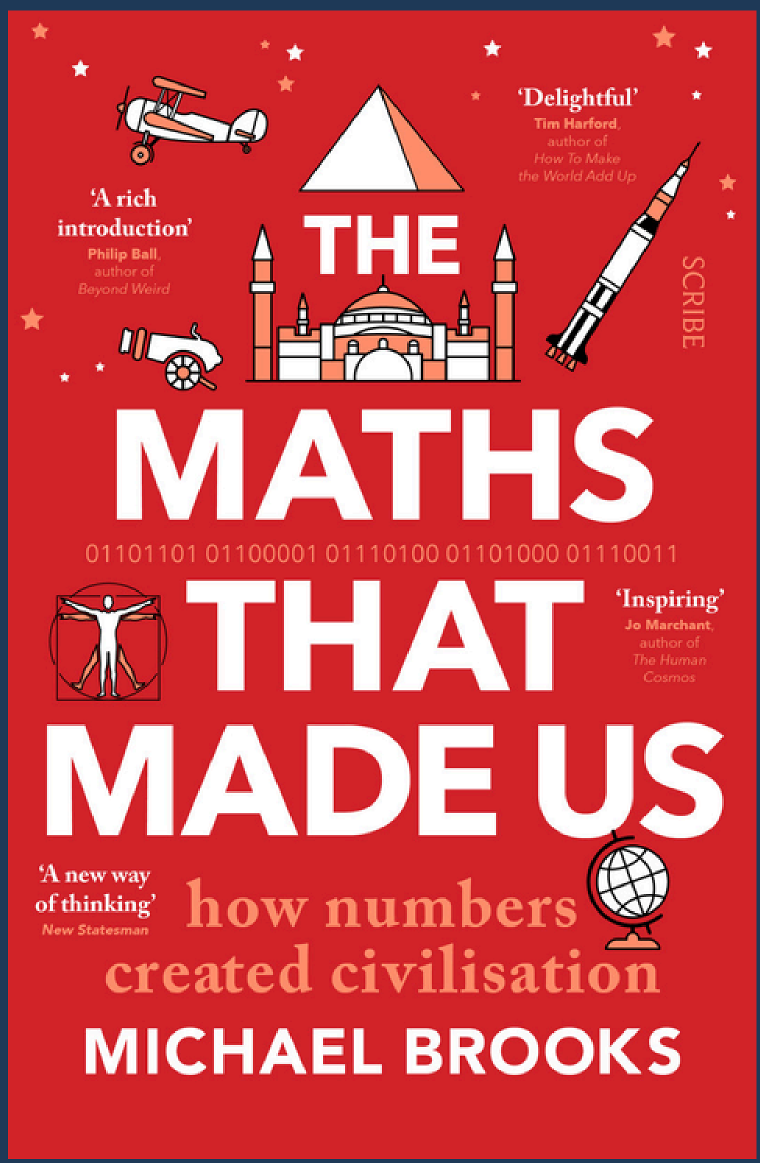
1 You have four squares that you can place on a large, flat table. You can place the squares so that their edges align, but their interiors cannot overlap. Your goal is to position the squares so that you can trace as many rectangles as possible using the edges of the squares. For example, if you had two squares instead of four, you could place the squares side by side, as shown below:

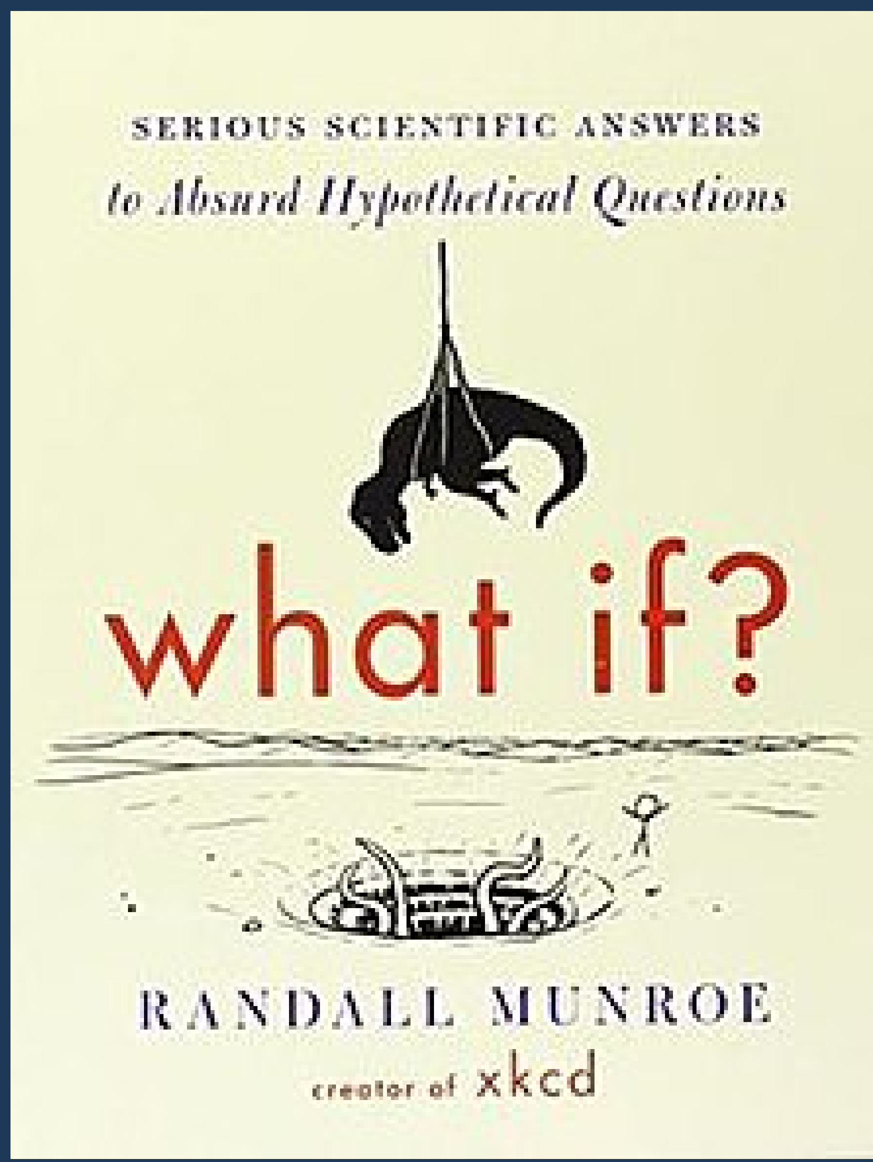


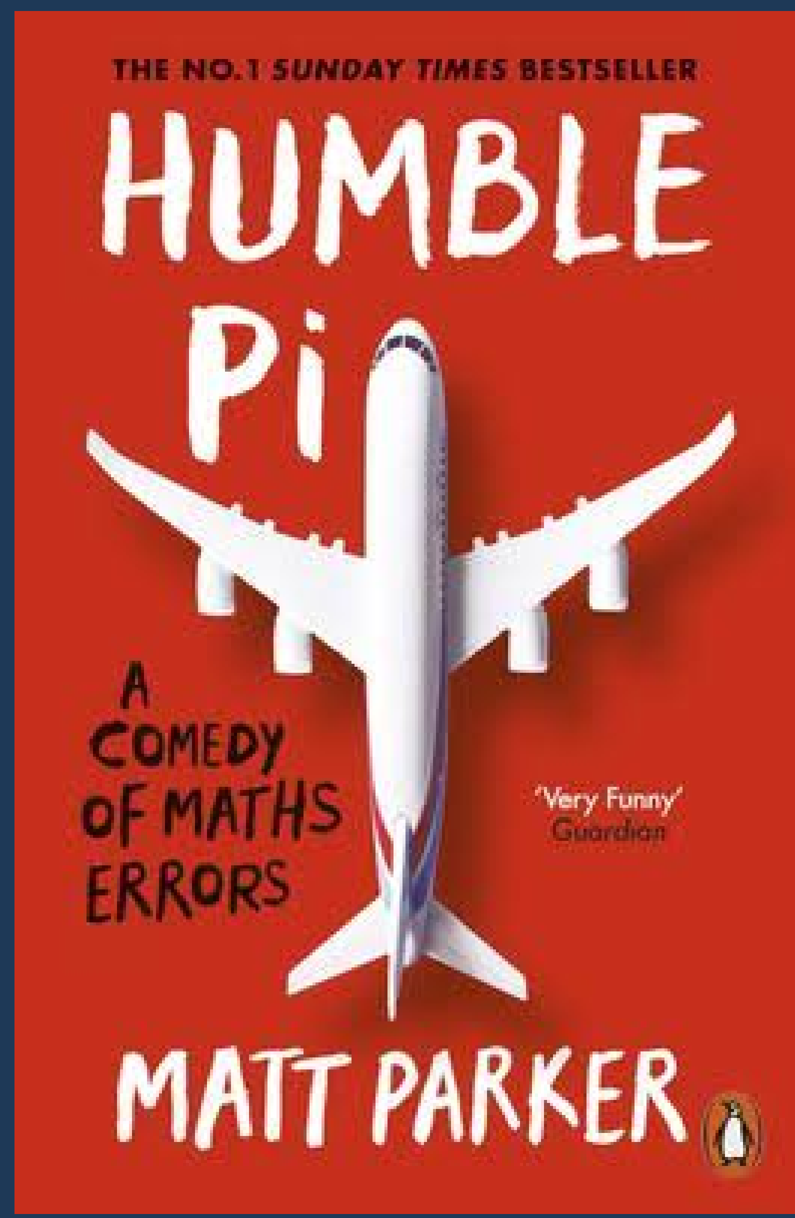
With this arrangement, it's possible to trace three rectangles: the square on the left, the square on the right and the larger rectangle around both squares. How would you arrange four squares to get as many rectangles as possible? And what is this number of rectangles?

2 There's a version of solitaire played in southern Italy with a deck of 40 Neapolitan cards, with four suits numbered from 1 to 10. The deck is shuffled and then cards are turned over one at a time. Flipping over the first card you say "one," the second card "two" and the third card "three." You repeat this, saying "one" for the fourth card, "two" for the fifth card and "three" for the sixth card. You continue your way through the deck, until you at last say "one" for the 40th card.

If at any point the number you say matches the value of the card you flip over, you lose.
What is your probability of winning the game?







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